(1) Prove the following Minimum Principle. If \( f \) is a non-constant analytic function on a bounded open set \( G \) and is continuous on the closure of \( G \), then either \( f \) has a zero in \( G \) or \( |f| \) assumes its minimum value on \( \partial G \).

(2) Let \( 0 < r < R \) and put \( A = \{ z : r \leq |z| \leq R \} \). Show that there is a positive number \( \varepsilon > 0 \) such that for each polynomial \( p(z) \in \mathbb{C}[z] \),

\[
\sup\{|p(z) - z^{-1}| : z \in A\} \geq \varepsilon
\]

That is, \( z^{-1} \) is not the uniform limit of polynomials on \( A \).

(3) Let \( f \) be analytic in the disk \( \Delta = \{ |z| < 1 \} \). We define \( A(r) = \max\{|\text{Re}f(z)| : |z| = r\} \) for \( 0 \leq r < 1 \). Show that unless \( f \) is a constant, \( A(r) \) is a strictly increasing function of \( r \).

(4) Let \( f \) be analytic on \( R_1 < |z| < R_2 \) which is not identically zero; define

\[
I_2(r) = \int_0^{2\pi} |f(re^{i\theta})|^2 d\theta
\]

Show that \( \log I_2(r) \) is a convex function of \( \log r \) for \( R_1 < r < R_2 \), i.e.,

\[
I_2(e^x)I_2(e^y) \geq \left(I_2(e^{(x+y)/2})\right)^2
\]

for all \( \log R_1 < x, y < \log R_2 \).

(5) A map \( f : X \to Y \) is proper if for every compact set \( D \subset Y \), \( f^{-1}(D) \) is compact. Show that if \( f : \mathbb{C} \to \mathbb{C} \) is proper and holomorphic, then \( f(z) \) must be a polynomial.

(6) Show that \( D_1 = \{ 0 < |z| < 1 \} \) and \( D_2 = \{ 1 < |z| < 2 \} \) are not biholomorphic, i.e., there does not exist a holomorphic map \( f : D_1 \to D_2 \) which is 1-1 and onto.

(7) Find a biholomorphic map \( f : \Delta = \{ |z| < 1 \} \to G = \{ |z| < 1, \text{Re}z > 0, \text{Im}z > 0 \} \).

(8) Find a holomorphic covering map \( f : \Delta = \{ |z| < 1 \} \to G = \{ r < |z| < R \} \) for some \( 0 < r < R \).