AA.1 Find a piecewise smooth closed curve $\gamma : [0, 1] \to \mathbb{C}$ such that $\mathbb{C} \setminus \gamma$ consists of three connected components $D_i$ for $i = 0, 1, 2$ such that the winding numbers of $\gamma$ are

$$
\nu(\gamma, z_0) = \begin{cases} 
0 & \text{for } z_0 \in D_0 \\
1 & \text{for } z_0 \in D_1 \\
2017 & \text{for } z_0 \in D_2 
\end{cases}
$$

AA.2 Let $D$ be a star-shaped open set in $\mathbb{C}$. Show that for any two continuous closed curves $\gamma_0 : [0, 1] \to D$ and $\gamma_1 : [0, 1] \to D$, there exists a continuous map $F : [0, 1] \times [0, 1] \to D$ such that

- $F(s, 0) = F(s, 1)$ for all $0 \leq s \leq 1$,
- $F(0, t) = \gamma_0(t)$ and $F(1, t) = \gamma_1(t)$ for all $0 \leq t \leq 1$.

AA.3 Let $f(z)$ be an analytic function on an open set $D$. Show that for every piecewise smooth closed curve $\gamma : [0, 1] \to D$ such that $f(z) \neq 0$ on $\gamma$,

$$
\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)} \, dz = \nu(f \circ \gamma, 0).
$$

AA.4 Use Rouché’s Theorem to determine the number of zeros of the following polynomials in $\{ |z| < 1 \}$:

(a) $2017z^{2017} + z + 1$;
(b) $z^{2017} + 2017z + 1$.

AA.5 Let $f(z)$ and $g(z)$ be two analytic functions on $\{ |z| < R \}$ for some $R > 1$. Show that if $|f(z)| < |g(z)|$ for all $|z| = 1$, then either $|f(z)| < |g(z)|$ for all $|z| \leq 1$ or there exists $z_0$ such that $|z_0| < 1$ and $f(z_0) = g(z_0)$.