

Sample Midterm II¹

- (1) (30 points) A lake has a volume of roughly 100 cubic miles and its equal inflow and out flow rates are 40 cubic miles per year. At year $t = 0$, a certain pollutant has a volume concentration of 0.05%, but after that the concentration of pollutant flowing into the lake drops to 0.01%. Answer the following questions, assuming the pollutant leaving the lake is well mixed with lake water.
 - (a) What is the IVP satisfied by the volume V (in cubic miles) of pollutant in the lake?
 - (b) What is the volume V of the pollutant in the lake at time t ?
 - (c) How long will it take to reduce the pollution concentration to 0.02% in volume?
- (2) (20 points) A cold beer with an initial temperature of 35°F warms up to 40°F in 10 minutes while sitting in a room of temperature 70°F. What will the temperature of the beer be after t minutes? After 20 minutes?
- (3) (20 points) You have an outstanding balance of \$2400 on your credit card and you keep spending \$100 each month. The annual interest rate is 12%. Suppose that you decide to pay it off in 2 years. (Use a continuous model.)
 - (a) How much should your monthly payment be?

¹<http://www.math.ucsb.edu/~xichen/math3c02s/p2.pdf>

(b) How much interest do you pay in total at last?

(4) (30 points) Solve the following differential equations.

(a)

$$\frac{dy}{dx} + e^x y = e^{2x}$$

with the initial condition $y(0) = 1$.

(b)

$$\frac{dy}{dx} + 3y = x + e^{2x}$$