Sample Midterm II\(^1\)

(1) (30 points) A lake has a volume of roughly 100 cubic miles and its equal inflow and outflow rates are 40 cubic miles per year. At year \(t = 0\), a certain pollutant has a volume concentration of 0.05\%, but after that the concentration of pollutant flowing into the lake drops to 0.01\%. Answer the following questions, assuming the pollutant leaving the lake is well mixed with lake water.

(a) What is the IVP satisfied by the volume \(V\) (in cubic miles) of pollutant in the lake?

(b) What is the volume \(V\) of the pollutant in the lake at time \(t\)?

(c) How long will it take to reduce the pollution concentration to 0.02\% in volume?

(2) (20 points) A cold beer with an initial temperature of 35\(^\circ\)F warms up to 40\(^\circ\)F in 10 minutes while sitting in a room of temperature 70\(^\circ\)F. What will the temperature of the beer be after \(t\) minutes? After 20 minutes?

(3) (20 points) You have an outstanding balance of $2400 on your credit card and you keep spending $100 each month. The annual interest rate is 12\%. Suppose that you decide to pay it off in 2 years. (Use a continuous model.)

(a) How much should your monthly payment be?

\(^1\text{http://www.math.ucsb.edu/~xichen/math3c02s/p2.pdf}\)
(b) How much interest do you pay in total at last?

(4) (30 points) Solve the following differential equations.

(a) \[
\frac{dy}{dx} + e^x y = e^{2x}
\]
with the initial condition \( y(0) = 1 \).

(b) \[
\frac{dy}{dx} + 3y = x + e^{2x}
\]