

PRINT NAME: _____

STUDENT ID NUMBER: _____

- (1) No books and notes are allowed.
- (2) You may use a scientific calculator and a notecard.
- (3) Write clearly and show your work in details.

Problem	Points	Score
1	30	
2	20	
3	20	
4	30	
Total	100	

You may need the following fact: the general solution for $y' = k(y + M)$ is $y = Ae^{kt} - M$, where k and M are constants.

(1) (30 points) A tank initially contains 200 gallons of fresh water, but then a salt solution of unknown concentration is poured into the tank at 2 gal/min. The well-stirred mixture flows out of the tank at the same rate. After 120 minutes, the concentration of salt in the tank is 1.4 lb/gal.

(a) (20 points) What is the concentration of the entering brine?

(b) (10 points) What is the concentration of salt in the tank after another 120 minutes?

- (2) (20 points) A cup of coffee initially at 200°F cools down to 120°F in a room at 70°F in 15 minutes. How long does it take for the coffee to cool down from 120°F to 90°F ? Use Newton's law of cooling.

- (3) (20 points) Suppose that a rich uncle has left you A_0 dollars, which is invested in an account with annual interest rate r . Show that if you make withdrawals amounting to d dollars per year (where $d > rA_0$), the time required to deplete the account to zero is

$$\frac{1}{r} \ln \left(\frac{d}{d - rA_0} \right)$$

years. Use a continuous model.

(4) (30 points) Solve the following differential equations.

(a) $\frac{dy}{dx} = \frac{2y}{x} + x^2 \cos x$

(b) $\frac{dy}{dt} + 2y = t$