CHEN	Math 3C Final	June 8, 2002

PRINT NAME:_____

PERM NUMBER:_____

(1) No books and notes are allowed.
(2) You may use a calculator and a notecard.
(3) Show your work in details.

Problem	Score	Problem	Score
1 (15)		2 (25)	
3 (20)		4 (40)	
5 (15)		6 (20)	
7 (20)		8 (20)	
9 (25)			
Total (200)			

(1) (15 points) Let $f(x, y) = xe^{xy} + ye^{x}$.

(a) (10 points) Find $\partial f/\partial x$ and $\partial f/\partial y$.

(b) (5 points) Find the tangent plane of the surface $z = f(x, y)$ at the point $(x, y) = (1, 1)$.

(2) (25 points) Consider the differential equation

$$\frac{dy}{dx} = y + x$$

with initial condition $y(1) = 1$.

(a) (10 points) Approximate $y(3)$ using Euler's method with stepsize $h = 2/3$.

(b) (10 points) Find the exact solution of the IVP.

(c) (5 points) Find the error in the approximation of part (a).

(3) (20 points) Consider the differential equation

$$\frac{dy}{dt} = y - 2y^3.$$

(a) (5 points) Find all the equilibrium solutions of the equation.

(b) (10 points) Sketch the direction field of the equation and determine the stabilities of the equilibrium solutions.

(c) (5 points) Let $y(t)$ be the solution of the equation with $y(0) = -1$. Find the limit $\lim_{t\to\infty} y(t)$. Justify your answer.

(4) (40 points) Solve the following differential equations.

(a) $\dfrac{dy}{dx} = \dfrac{\ln x}{\ln y}$.

(b) $\dfrac{dy}{dx} = e^x(e^x - y)$ with $y(0) = 1$.

(c) $\dfrac{dy}{dt} + \dfrac{2t+1}{t} y = 2.$

(d) $xy' + y = xy$ with $y(2) = 1.$

(5) (15 points) Find the second (degree 2) Taylor polynomial of $\tan x$ at $x = \pi/4$.

(6) (20 points) Let

$$f(x) = \frac{1+x}{1+x^2}.$$

(a) (10 points) Find the Taylor series of $f(x)$ at $x = 0$.

(b) (10 points) What is the coefficient of x^{2002} in the Taylor series of $f(x)$ at $x = 0$?

(7) (20 points)You have an outstanding balance of $2400 on your credit card and you keep spending $100 each month. The annual interest rate is 12%. Suppose that you decide to pay it off in 2 years. (Use a continuous model.)

(a) How much should your monthly payment be?

(b) How much interest do you pay in total at last?

(8) (20 points) Into a tank containing 100 gal of fresh water, Jane was to have added 10 lbs of salt but accidentally added 20 lbs instead. To correct her mistake she started adding fresh water at a rate of 3 gal/min, while drawing off well mixed solution at the same rate. How long will it take until the tank contains the correct amount of salt?

(9) (25 points) A population grows according to the logistic law with a limiting population of 5×10^9 individuals. The initial population of 10^9 begins growing by doubling every hour. What will the population be after 4 hours?