

PRINT NAME: _____

PERM NUMBER: _____

DISCUSSION SECTION AND TA'S NAME: _____

Problem	Score	Problem	Score
1 (25)		2 (15)	
3 (15)		4 (10)	
5 (20)		6 (40)	
7 (15)		8 (30)	
9 (40)		10 (10)	
Total (220)			

1. (25 points)

(a) (5 points) Identify which of the following differential equations are autonomous by circling your choices.

i. $x' = \cos x$

ii. $p' = p(p - 2)(3 - p)$

iii. $x' = \sin t$

iv. $y' = 3y + 5$

v. $y' = 3y + 5t$

(b) (10 points) Sketch the direction fields for (i) and (iii) in part (a).

(c) (10 points) In the direction fields obtained above, sketch the solution curve in each that corresponds to the initial condition $x(0) = 1$.

2. (15 points) Let $f(x, y) = xe^{-y} + 2ye^x$.

(a) (10 points) Find $\partial f/\partial x$ and $\partial f/\partial y$.

(b) (5 points) Find the tangent plane to the surface $z = f(x, y)$ at $(0, 0, 0)$.

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- (15 points) Use Euler's method with stepsize $h = 1$ to approximate $x(4)$ for the solution $x(t)$ of the differential equation $x' = xt$ with initial condition $x(0) = 7$.

4. (10 points) Find the 2nd degree Taylor polynomial for $f(x) = \tan x$ at $x = \pi/4$.

5. (20 points) Let $x' = x - x^2$.

(a) (10 points) Find the explicit solution for the differential equation that satisfies the initial condition $x(0) = 2$.

(b) (5 points) Explain why the graph of the solution $x(t)$ obtained in part (a) does not cross the t -axis in the tx plane.

(c) (5 points) Find $\lim_{t \rightarrow \infty} x(t)$ for the solution $x(t)$ you found in part (a).

6. (40 points) Suppose you deposit \$2000 into a bank account with a 6% annual interest rate.
- (a) (5 points) Find the formula for the account balance if interest is compounded monthly. (Your formula gives the account balance at the end of t -th month.)
- (b) (5 points) Find the formula for the account balance if interest is compounded continuously. (Your formula gives the account balance at the end of t -th month.)
- (c) (10 points) Suppose you withdraw \$100 a month. What is the differential equation modelling this situation?

(d) (10 points) Solve the equation you got in part (c).

(e) (10 points) When will you run out of money?

7. (15 points) Find the interval of existence of the solution for

$$\frac{dx}{dt} = \frac{xt}{t^2 - 1}$$

with $x(0) = 1$.

8. (30 points) For each of the following differential equations, do: (i) solve the equation (ii) check your solution (iii) identify the steady-state and transient parts of your solution.

(a) (15 points) $x' + 2x = t$ with $x(0) = 1$.

(b) (15 points) $x' + \frac{3x}{t} = t^3$ with $x(1) = 1$

9. (40 points) Consider the differential equation $x' = \cos x$. Limit your discussion to $0 \leq x \leq 2\pi$ in the following questions.

(a) (10 points) Find all equilibrium solutions to the given differential equation.

(b) (10 points) Show the phase diagram for the differential equation and determine the stability for each equilibrium.

(c) (10 points) Find the linearization of the differential equation at an equilibrium solution. (You may pick any equilibrium solution of the equation as you like.)

(d) (10 points) Solve the linearized differential equation you obtained in part (c).

10. (10 points) Find the Taylor series and its radius of convergence for the function $f(x) = e^x - e^{-x}$ at $x = 0$.