

Sample Midterm II

1. (30 points) Solve the following differential equations.
 - (a) (15 points) $x' + tx = t$ with $x(0) = 0$.
 - (b) (15 points) $x' + x = e^t$.
2. (10 points) The half-life of Oxygen 14 is 70.6s. How long does it take for a 10 g mass to decay by 1 g?
3. (25 points) Suppose you want to purchase a car that costs 17,500 dollars. A generous relative contributes 12,500 dollars but you have to save for the rest. If you can save 150 dollars a month and the bank offers a 9% interest rate. Assume that the interest and deposits are added continuously.
 - (a) (20 points) How long do you have to wait until you can buy the car? (Note that you are supposed to put the 12,500 dollars in the bank at the beginning.)
 - (b) (5 points) How much interest do you earn in total at the end?
4. (35 points) Consider the differential equation $x' + x = mt$, where m is any constant.
 - (a) (15 points) Find the solution that satisfies the initial condition $x(0) = x_0$. (Express the solution in terms of m and x_0 .)
 - (b) (10 points) Suppose that the actual initial condition is $x_0 = 1$ and the measured initial condition is $\hat{x}_0 = 2$. Find the absolute error E_a and the relative error E_r in the solution.
 - (c) (10 points) Find $\lim_{t \rightarrow \infty} E_a$ and $\lim_{t \rightarrow \infty} E_r$.