Sample Midterm I

1. (20 points) Let \( f(x, y) = x + y^2 \).
   (a) (5 points) Draw the level curves of \( f(x, y) \).
   (b) (10 points) Find \( \partial f / \partial x \) and \( \partial f / \partial y \).
   (c) (5 points) Find the tangent plane of the surface \( z = f(x, y) \)
      at the point \((1, 1, 2)\).

2. (40 points) Consider the following differential equation
   \[ \frac{dp}{dt} = 1 - e^p. \]
   (a) (10 points) Draw the direction field.
   (b) (10 points) Find the general solution.
   (c) (5 points) Let \( p(0) = p_0 \). Suppose that \( p_0 < 0 \). Find \( \lim_{t \to \infty} p(t) \).
   (d) (5 points) Let \( p(0) = p_0 \). Suppose that \( p_0 > 0 \). Find \( \lim_{t \to \infty} p(t) \).
   (e) (10 points) Find all the equilibrium solutions of the equation
      and determine their stabilities.

3. (20 points) Consider the following differential equation
   \[ \frac{dy}{dt} = y - t \]
   with initial condition \( y(1) = 0 \).
   (a) (10 points) Use Euler’s method to approximate \( y(4) \) by taking
      the step \( h = 1 \).
   (b) (5 points) Verify that \( y(t) = Ae^t + t + 1 \) is the general solution
      of the equation, where \( A \) is a constant.
   (c) (5 points) What is the absolute error in the approximation of
      part (a)\

4. (20 points) Solve the following differential equations.
   (a) (10 points)
   \[ \frac{dy}{dx} = \frac{\ln x}{\ln y} \]
   (b) (10 points)
   \[ xy' + y = xy \]
   with initial condition \( y(2) = 0 \).