

PRINT NAME: \_\_\_\_\_

PERM NUMBER: \_\_\_\_\_

DISCUSSION SECTION AND TA'S NAME: \_\_\_\_\_

Problem	Points	Score
1	40	
2	20	
3	20	
4	20	
Total	100	

1. (40 points) Consider the following differential equation

$$\frac{dp}{dt} = p(p - 2).$$

- (a) (5 points) Draw the direction field.
- (b) (5 points) Find all the equilibrium solutions of the equation and determine their stabilities.

- (c) (10 points) Find the general solution. (You must express  $p$  in terms of  $t$  in your final answer.)

(d) (10 points) Let  $p(t)$  be the solution of the equation satisfying the initial condition  $p(0) = -4$ . Find  $\lim_{t \rightarrow \infty} p(t)$ . You must justify your answer.

(e) (10 points) Let  $p(t)$  be the solution of the equation satisfying the initial condition  $p(0) = 4$ . Find  $\lim_{t \rightarrow -\infty} p(t)$ . You must justify your answer.

2. (20 points) Consider the following differential equation

$$\frac{dy}{dt} = 1 - y$$

with initial condition  $y(-1) = 1$ .

- (a) (10 points) Use Euler's method to approximate  $y(2)$  by taking the stepsize  $h = 1$ .

(b) (5 points) Find the real solution of the equation with the initial condition  $y(-1) = 1$ .

(c) (5 points) What is the absolute error in the approximation of part (a)?

3. (20 points) Solve the following differential equations.

(a) (10 points)

$$\frac{dy}{dx} = \frac{x}{y}$$

with initial condition  $y(1) = 2$ .

(b) (10 points)

$$\frac{dy}{dx} = xe^{y-x}$$

with initial condition  $y(0) = 1$ .

4. (20 points) Let  $f(x, y) = \sqrt{xy}$ .

(a) (5 points) Draw the level curves of  $f(x, y)$ . (Show me at least three level curves.)

(b) (10 points) Compute  $\partial f/\partial x$  and  $\partial f/\partial y$ .

(c) (5 points) Find the tangent plane of  $z = f(x, y)$  at the point  $(x, y) = (4, 1)$ .