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1. (40 points) Consider the following differential equation

\[ \frac{dp}{dt} = p(p - 2). \]

(a) (5 points) Draw the direction field.

(b) (5 points) Find all the equilibrium solutions of the equation and determine their stabilities.
(c) (10 points) Find the general solution. (You must express $p$ in terms of $t$ in your final answer.)
(d) (10 points) Let $p(t)$ be the solution of the equation satisfying the initial condition $p(0) = -4$. Find \( \lim_{t \to -\infty} p(t) \). You must justify your answer.

(e) (10 points) Let $p(t)$ be the solution of the equation satisfying the initial condition $p(0) = 4$. Find \( \lim_{t \to -\infty} p(t) \). You must justify your answer.
2. (20 points) Consider the following differential equation

\[
\frac{dy}{dt} = 1 - y
\]

with initial condition \(y(-1) = 1\).

(a) (10 points) Use Euler’s method to approximate \(y(2)\) by taking the stepsize \(h = 1\).
(b) (5 points) Find the real solution of the equation with the initial condition $y(-1) = 1$.

(c) (5 points) What is the absolute error in the approximation of part (a)?
3. (20 points) Solve the following differential equations.

(a) (10 points)

\[ \frac{dy}{dx} = \frac{x}{y} \]

with initial condition \( y(1) = 2 \).
(b) (10 points)

\[ \frac{dy}{dx} = xe^{y-x} \]

with initial condition \( y(0) = 1 \).
4. (20 points) Let \( f(x, y) = \sqrt{xy} \).

(a) (5 points) Draw the level curves of \( f(x, y) \). (Show me at least three level curves.)

(b) (10 points) Compute \( \partial f / \partial x \) and \( \partial f / \partial y \).

(c) (5 points) Find the tangent plane of \( z = f(x, y) \) at the point \( (x, y) = (4, 1) \).