

Sample Midterm II

- (1) (30 points) Solve the following differential equations.
 - (a) (10 points) $x' + x = e^t$.
 - (b) (10 points) $x' + tx = t$ with $x(0) = 0$.
 - (c) (10 points) $tx' + x = \sin t$.
- (2) (10 points) The half-life of Oxygen 14 is 70.6s. How long does it take for a 10 g mass to decay by 1 g?
- (3) (15 points) Suppose a cup of hot tea initially at 100 °C cools to 80 °C after 30 minutes under the room temperature at 30 °C. How long does it take for the tea to cool down another 20 °C? (Use Newton's cooling law.)
- (4) (45 points) Consider the differential equation $x' + x = mt$, where m is any constant.
 - (a) (15 points) Find the solution that satisfies the initial condition $x(0) = x_0$. (Express the solution in terms of m and x_0 .)
 - (b) (10 points) Suppose that the actual initial condition is $x_0 = 1$ and the measured initial condition is $\hat{x}_0 = 2$. Find the absolute and relative errors in the solution. (Your answers should involve m and t .)
 - (c) (20 points) Describe the qualitative properties of the solution $x(t)$ as t becomes large. Specifically, answer the following questions. You have to justify your answer.
 - (i) Does $x(t)$ approach a straight line (asymptote)? If so, what is the slope?
 - (ii) For what values of m and x_0 , $\lim_{t \rightarrow \infty} x(t) = 0$?