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1. (30 points) Consider the following differential equation

\[ \frac{dp}{dt} = 2p - p^2. \]

(a) (10 points) Find the general solution.

(b) (5 points) Let \( p(0) = p_0 \). Suppose that \( 0 < p_0 < 2 \). Find \( \lim_{t \to \infty} p(t) \).

(c) (5 points) Let \( p(0) = p_0 \). Suppose that \( p_0 > 2 \). Find \( \lim_{t \to \infty} p(t) \).

(d) (10 points) Find all the equilibrium solutions of the equation and determine their stabilities.
2. (20 points) Consider the following differential equation

\[
\frac{dy}{dt} = y + t^2
\]

with initial condition \( y(1) = 1 \).

(a) (10 points) Use Euler’s method to approximate \( y(3) \) by taking the step \( h = 1 \).

(b) (5 points) Verify that \( y(t) = Ae^t - t^2 - 2t - 2 \) is the general solution of the equation, where \( A \) is a constant.

(c) (5 points) What is the absolute error in the approximation of part (a)?
3. (20 points) Let $f(x, y) = \sin x + \cos y$.

(a) (10 points) Compute $\partial f / \partial x$ and $\partial f / \partial y$.

(b) (10 points) Find the tangent plane of $z = f(x, y)$ at the point $(0, \pi/2, 0)$. 

4. (20 points) Solve the following differential equations.

(a) (10 points)

\[ \frac{dy}{dx} = x(y^2 + 1). \]

(b) (10 points)

\[ \frac{dy}{dx} = e^y \]

with initial condition \( y(0) = 0 \).
5. (10 points) Find all the constant solutions of the differential equation

\[ \frac{dy}{dx} = x^2 y + y^2. \]

You must justify your answer.