PRINT NAME:________________________________________

PERM NUMBER:______________________________________

DISCUSSION SECTION AND TA'S NAME:______________________

| Problem | Score | Problem | Score |
|---|---|---|---|
| 1 (15) | | 2 (15) | |
| 3 (10) | | 4 (45) | |
| 5 (15) | | 6 (20) | |
| 7 (20) | | 8 (15) | |
| 9 (25) | | 10 (20) | |
| Total (200) | | | |

1. (15 points) Solve the following differential equations.

(a) (5 points)

$$\frac{dx}{dt} = t^3$$

(b) (5 points)

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

(c) (5 points)

$$\frac{dy}{dx} = \frac{1}{x}$$

2. (15 points) Compute the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ of the function $f(x,y) = x^2y^3$ and find the equation for the tangent plane to the surface defined by $z = f(x,y)$ at the point $(2,-1)$.

3. (10 points) Match the direction fields in Figures 1 and 2 and the differential equations listed below. Explain which vector field could possibly be a model for the dynamics of a population and why the other one is not realistic. (Figure 2 is on the next page.)

   (a) $p' = p(p-2)(5-p)$

   (b) $p' = t - p - 2$

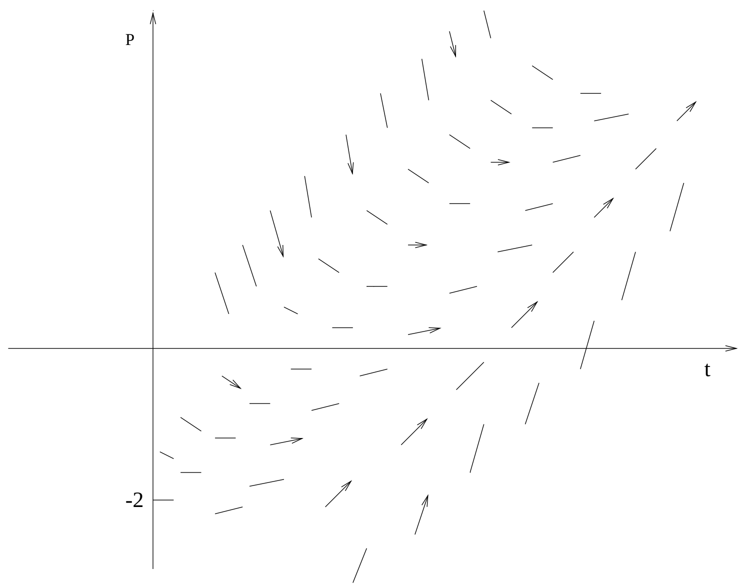


Figure 1: The direction field.

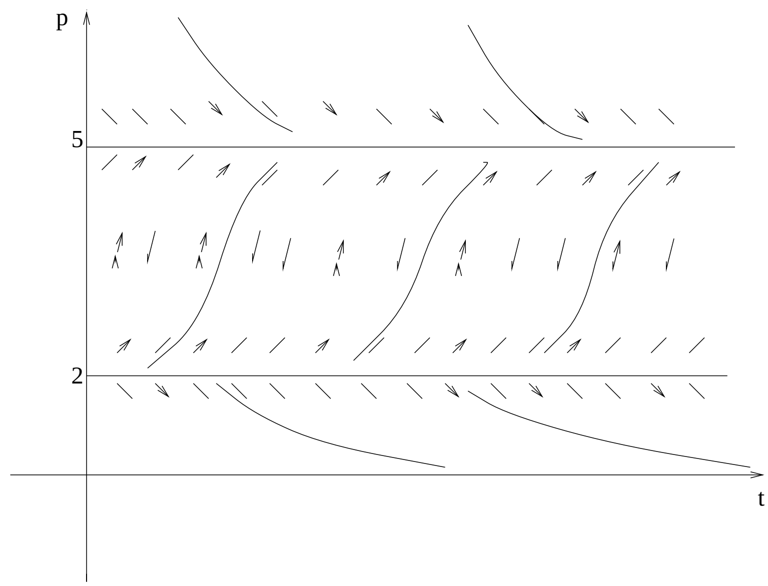


Figure 2: The direction field.

4. (45 points) The general logistic equation can be written in the form

$$\frac{dp}{dt} = ap - bp^2$$

where $a$ and $b$ are positive constants.

(a) (10 points) Find the equilibrium solutions of this equation and give them a biological interpretation if that is possible.

(b) (10 points) Determine the stability of the equilibrium solutions.

(c) (5 points) Draw the direction field (in $t$ and $p$ plane).

(d) (10 points) Determine what happens to populations with initial values, $p(0) = a/2b$, $p(0) = 3a/2b$, as $t \to \infty$.

(e) (10 points) Compute $p''$ and find the inflection points of $p(t)$. (Express $p''$ in terms of $p$ and your answer should not involve $p'$. An inflection point of $p(t)$ is a point where $p''(t)$ vanishes.)

5. (15 points) Use stepsize $h = 0.2$ and Euler's Method to find the solution of the differential equation

$$\frac{dy}{dx} = -y + x,\ y(0) = 1,$$

at $x = 0.8$.

6. (20 points) Solve the following differential equations

(a) (10 points)

$$\frac{dx}{dt} = (1+x^2)\cos(t), \;\; x(0) = 0$$

(b) (10 points)

$$\frac{dx}{dt} = x^{1/4}t^3, \;\; x(0) = 1.$$

7. (20 points) Suppose you want to purchase a car that costs $17,500$ dollars. A generous relative contributes $12,500$ dollars but you have to save for the rest. If you can save 150 dollars a month and the bank offers a 9% interest rate, that is compounded continuously, how long do you have to wait until you can buy the car? Make sure that your answer makes sense. (Note that you are supposed to put the $12,500$ dollars you received from your relative in the bank at the beginning.)

8. (15 points) Solve the initial value problem and find the transient and the steady-state parts of the solution.

$$\frac{dx}{dt} + 2x = t, \qquad x(0) = 3.$$

9. (25 points) Consider the logistic equation

$$\frac{dp}{dt} = 3p - 2p^2$$

(a) (10 points) Linearize the equation at $p = 0$ and $p = 3/2$.

(b) (15 points) Suppose that the population is close to $p = 3/2$, but we make a small absolute error $\varepsilon = \hat{p}_0 - p_0$, when we measure it at $t = t_0$. Use the equation linearized at $p = 3/2$ to estimate the absolute error $|\hat{p}(t) - p(t)|$.

10. (20 points) Compute the Taylor series of the functions below and find their radius of convergence.

   (a) (10 points) $f(x) = (1-x^2)^{-1}$ at $x = 0$.

   (b) (10 points) $f(x) = x\sin(x) - \cos(x)$ at $x = 0$.