

1. Since $s'(t) = v(t)$, $s(t) = \int v(t)dt = t^4 - 4t + C$. By $s(0) = 1$, $C = 1$. So $s(t) = t^4 - 4t + 1$. The coordinates of the particle after 3 and 10 seconds are $s(3) = 70$ and $s(10) = 9961$, respectively.

2. (a) Right endpoints: $2(4 + 16 + 36) = 112$

(b) Left endpoints: $2(0 + 4 + 16) = 40$

(c) Midpoints: $2(1 + 9 + 25) = 70$

If $f(x)$ is an increasing function, (a) is an overestimate, (b) is an underestimate and (c) could be an overestimate or underestimate depending on the function.

3. (a) By the fundamental theorem of calculus, $f'(t) = e^{t^2}$.

(b) Let $F(t)$ be an antiderivative of e^{t^2} . Then by the fundamental theorem of calculus, $f(t) = F(t^2) - F(0)$. So $f'(t) = F'(t^2)(t^2)' = 2te^{t^4}$.

4. (a)

$$\begin{aligned} \int_1^2 \frac{(1+x)^3}{x} dx &= \int_1^2 \frac{1+3x+3x^2+x^3}{x} dx \\ &= \int_1^2 \left(\frac{1}{x} + 3 + 3x + x^2 \right) dx \\ &= \left(\ln|x| + 3x + \frac{3x^2}{2} + \frac{x^3}{3} \right) \Big|_1^2 \\ &= \ln 2 + 3 + \frac{3}{2} + \frac{7}{3} = \frac{41}{6} + \ln 2 \end{aligned}$$

(b) Substitute $u = x^2 + 1$:

$$\int_1^2 x\sqrt{x^2+1} dx = \frac{1}{2} \int_2^5 \sqrt{u} du = \frac{1}{3}(5\sqrt{5} - 2\sqrt{2}).$$

(c)

$$\int \frac{1+x}{1+x^2} dx = \int \frac{1}{x^2+1} dx + \int \frac{x}{x^2+1} dx = \tan^{-1} x + \frac{1}{2} \ln(x^2+1) + C.$$

(d) Substitute $u = 1 + e^x$:

$$\int \frac{1}{u} du = \ln|u| + C = \ln|1 + e^x| + C = \ln(1 + e^x) + C.$$