Review for Midterm I

Here are a list of things you need to know:
1. antiderivatives (indefinite integrals); table on p. 402;
2. Riemann sum; approximation of integrals with left and right endpoints and midpoints;
3. fundamental theorem of calculus (including total change theorem);
4. substitution rule.

A Practice Midterm

1. (20 points) A particle is moving along a line so that its velocity is given by
\[ v(t) = 4t^3 - 4 \]
where \( t \) is measured in seconds and \( v(t) \) is measured in meters per second. Let \( s(t) \) be the position function of the particle and \( s(0) = 1 \).

(a) (5 points) Find the coordinate of the particle after 3 seconds.
(b) (5 points) Find the coordinate of the particle after 10 seconds.
(c) (10 points) Find the coordinate of the particle after \( t \) seconds.

2. (20 points) The following table gives the value of a function \( f(x) \) obtained from an experiment.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
</tr>
</tbody>
</table>

\[^1\text{http://www.math.ucsb.edu/~xichen/math3b02w/p1.pdf} \]
Estimate $\int_0^6 f(x)\,dx$ using three equal intervals with (a) right endpoints (b) left endpoints (c) midpoints. If $f(x)$ is known to be an increasing function on $[0,6]$, can you say whether your estimates are less than or greater than the exact value of the integral.

3. (20 points)
   (a) (10 points) Let $f(t) = \int_0^t e^{x^2} \,dx$. Find $f'(t)$.
   (b) (10 points) Let $f(t) = \int_0^{t^2} e^{x^2} \,dx$. Find $f'(t)$.

4. (40 points) Evaluate the following integrals.
   (a) (10 points) $\int_1^2 \frac{(1 + x)^3}{x} \,dx$
   (b) (10 points) $\int_1^2 x \sqrt{x^2 + 1} \,dx$
   (c) (10 points) $\int_1^1 \frac{1 + x}{1 + x^2} \,dx$
   (d) (10 points) $\int_0^1 \frac{e^x}{1 + e^x} \,dx$