

A Sample Final¹

- (1) (10 points) Let $v(t) = t + t^2$ be the velocity of a particle moving a line with t measured in seconds and $v(t)$ measured in meters per second. Find the distance traveled by the particle from $t = 1$ to $t = 2$.
- (2) (50 points) Evaluate the following integrals.
- (a) (10 points) $\int \frac{dx}{e^x + 1}$
- (b) (10 points) $\int_0^{\pi/4} \sin^3 x \cos^2 x dx$
- (c) (10 points) $\int x^2 e^x dx$
- (d) (10 points) $\int \frac{\sqrt{x}}{1 + \sqrt{x}} dx$
- (e) (10 points) $\int \frac{x^3}{x^2 + 4x + 4} dx$
- (3) (20 points) Use (a) Trapezoidal Rule and (b) Simpson's Rule to approximate the integral $\int_0^2 \sqrt{x^3 + 1} dx$ by dividing $[0, 2]$ into eight subintervals. Write down the expressions and do not evaluate.
- (4) (20 points) Explain why the integral $\int_0^\infty x e^{-x} dx$ is improper and evaluate its value if it converges.
- (5) (60 points) Let C be the curve given by $y = e^x$ with $0 \leq x \leq 1$ and R be the region below the curve.
- (a) (10 points) Find the area of R .
- (b) (10 points) Find the volume of the solid obtained by rotating R around the x -axis.
- (c) (10 points) Find the volume of the solid obtained by rotating R around the y -axis.
- (d) (10 points) Find the length of C . Set up the integral and do not evaluate.
- (e) (10 points) Find the area of the surface obtained by rotating C around x -axis. Set up the integral and do not evaluate.
- (f) (10 points) Find the area of the surface obtained by rotating C around y -axis. Set up the integral and do not evaluate.
- (6) (20 points) Find the Cartesian equations of the following parametric curves.
- (a) (10 points) $x = 2t + 4, y = t - 1$
- (b) (10 points) $x = \sqrt{t}, y = 1 - t$
- (7) (20 points) If the work required to stretch a spring 1 ft beyond its natural length is 12 ft-lb, how much work is needed to stretch it 9 in beyond its natural length?

¹<http://www.math.ucsb.edu/~xichen/math3b02w/f.pdf>