A Sample Midterm for Midterm II

(1) (15 points) Estimate \( \int_0^{2\pi} \sin(x + \pi/4) \, dx \) using
   (a) (5 points) Midpoint Rule with \( n = 4 \);
   (b) (5 points) Trapezoidal Rule with \( n = 4 \);
   (c) (5 points) Simpson’s Rule with \( n = 4 \).

(2) (15 points) Determine whether the following improper integrals are convergent or divergent. You must justify your answers.
   (a) (5 points) \( \int_0^1 x^{-3/2} \sin x \, dx \)
   (b) (5 points) \( \int_1^\infty \frac{dx}{e^x + x^2} \)
   (c) (5 points) \( \int_1^\infty \frac{dx}{\sqrt{x^4 + 1}} \)

(3) (40 points) Let \( R \) be the region bounded by the curves \( x = e^y \), \( x = 0 \), \( y = 0 \) and \( y = 1 \).
   (a) (10 points) Let \( S_1 \) be the solid obtained by rotating \( R \) around the \( x \)-axis. Find the volume of \( S_1 \).
   (b) (10 points) Let \( S_2 \) be the solid obtained by rotating \( R \) around the \( y \)-axis. Find the volume of \( S_2 \).
   (c) (20 points) A tank full of water has the shape of \( S_2 \) as in part (b) (the top of the tank is on the line \( y = 1 \) and its bottom is on \( y = 0 \); all coordinates are measured in meters). Find the work required to pump the water out of the tank. (Use the fact that the density of water is 1000 kg/m\(^3\) and take \( g = 10 \) m/s\(^2\) to make your computation simple.)

(4) (30 points) Evaluate the following integrals.
   (a) (5 points) \( \int e^t - 1 \, dt \)
   (b) (5 points) \( \int_0^{\pi/4} (\sin 5x)(\sin 2x) \, dx \)
   (c) (5 points) \( \int \frac{dx}{x\sqrt{x^2 + 3}} \)

\(^1\)http://www.math.ucsb.edu/~xichen/math3b00w/p2.pdf
(d) (5 points) \[ \int x^3 e^x \, dx \]

(e) (10 points) \[ \int \frac{1}{x^4 - 1} \, dx \]

**A Reminder for Time & Location of the Final:** Mar. 20, 7:30 - 10:30pm, Phelps 1260