(1) (a) $2(4+16+20)=80$
(b) $2(0+4+16)=40$
(c) $2(1+9+25)=70$

If $f(x)$ is an increasing function, (a) is an overestimate, (b) is an underestimate and (c) could be an overestimate or underestimate depending on the function.

(2)

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\tan\left(\frac{k\pi}{4n}\right) = \frac{4}{\pi}\int_0^{\pi/4}\tan x dx$$
$$= -\frac{4}{\pi}\ln|\cos x|\Big]_0^{\pi/4}$$
$$= \frac{2\ln 2}{\pi}$$

(3) Suppose that $F(t)$ is an antiderivative of e^{t^2}. Then by Fundamental Theorem of Calculus, $f(x) = F(3x) - F(2x)$. So $f'(x) = 3F'(3x) - 2F'(2x) = 3e^{9x^2} - 2e^{4x^2}$.

(4) (a) The area of R is

$$\int_0^2 |x-x^2|dx = \int_0^1 (x-x^2)dx + \int_1^2 (x^2-x)dx = 1.$$

(b) The volume of the solid obtained by revolving R around the x-axis is

$$\pi\int_0^1 (x^2-x^4)dx + \pi\int_1^2 (x^4-x^2)dx = 4\pi.$$

(c) The volume of the solid obtained by revolving R around the y-axis is

$$\pi\int_0^1 ((\sqrt{y})^2-y^2)dy + \pi\int_1^2 (y^2-(\sqrt{y})^2)dy + \pi\int_2^4 (2^2-(\sqrt{y})^2)dy = 3\pi.$$

(5) (a)

$$\int 2^x dx = \int e^{(\ln 2)x}dx = \frac{e^{(\ln 2)x}}{\ln 2} + C = \frac{2^x}{\ln 2} + C.$$

(b) Substitute $u = x^2+1$

$$\int_1^2 x\sqrt{x^2+1}dx = \frac{1}{2}\int_2^5 \sqrt{u}du = \frac{1}{3}(5\sqrt{5}-2\sqrt{2}).$$

(c)

$$\int \frac{1+x}{1+x^2}dx = \int \frac{1}{x^2+1}dx + \int \frac{x}{x^2+1}dx = \tan^{-1} x + \frac{1}{2}\ln(x^2+1) + C.$$

(d) Substitue $u = \sqrt{x-1}$

$$\int_1^2 x\sqrt{x-1}dx = 2\int_0^1 (u^2+1)u^2 du = \frac{16}{15}.$$

Solution of Problem 70 on page 358 (This is the only homework problem someone asked me about. So I assume that you had no trouble with the rest)

Let $f(x)$ be the mass of the part of the rod x centimeter measured from one end of the rod. Then $f'(x) = \rho(x) = 1/\sqrt{x}$. So $f(x) = 2\sqrt{x} + C$. Since $f(0) = 0$, $C = 0$. So $f(x) = 2\sqrt{x}$. The mass of the rod is $f(100) = 20$ g.