

1. (20 points) The following table gives the value of a function  $f(x)$  obtained from an experiment.

$x$	0	1	2	3	4	5	6
$f(x)$	0	1	4	9	16	25	36

Estimate  $\int_0^6 f(x)dx$  using three equal subintervals with (a) right endpoints, (b) left endpoints and (c) midpoints. If the function is known to be an increasing function, can you say whether your estimates are less than or greater than the exact value of the integral?

2. (20 points) Evaluate the limit

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \tan\left(\frac{k\pi}{4n}\right)$$

by expressing it as an integral.

3. (10 points) Let

$$f(x) = \int_{2x}^{3x} e^{t^2} dt.$$

Find  $f'(x)$ .

4. (30 points) Let  $R$  be the region between the curves  $y = x$  and  $y = x^2$  from  $x = 0$  to  $x = 2$ . Do the following.
- (a) (10 points) Find the area of  $R$ .
  - (b) (10 points) Find the volume of the solid obtained by revolving  $R$  around the  $x$ -axis.
  - (c) (10 points) Find the volume of the solid obtained by revolving  $R$  around the  $y$ -axis.
5. (20 points) Evaluate the following integrals.

(a) (5 points)  $\int 2^x dx$

(b) (5 points)  $\int_1^2 x\sqrt{x^2+1} dx$

(c) (5 points)  $\int \frac{1+x}{1+x^2} dx$

(d) (5 points)  $\int_1^2 x\sqrt{x-1} dx$