# Solution for Midterm I [1]

(1) (a) $2(-2-14-34) = -100$
(b) $2(2-2-14) = -28$
(c) $2(1-7-23) = -58$
Since $2-x^2$ is a decreasing function on $[0,6]$, (a) is an underestimate and (b) is an overestimate. Since $2-x^2$ is concave downward, (c) is an overestimate. Or you can just compare (a), (b) and (c) with the exact value of $\int_0^6 (2-x^2)dx = (2x - x^3/3)]_0^6 = -60$.

(2)

$$\lim_{n\to\infty} \sum_{k=n+1}^{2n} \frac{n}{k^2} = \lim_{n\to\infty} \frac{1}{n} \sum_{k=n+1}^{2n} \frac{1}{\left(\frac{k}{n}\right)^2}$$
$$= \int_1^2 \frac{1}{x^2} dx = -\frac{1}{x}\Big]_1^2 = \frac{1}{2}.$$

(3) Since $F'(x) = f(x)$,

$$F''(x) = f'(x) = \frac{d}{dx}\left(\int_1^{x^2} \frac{\sqrt{1+u^4}}{u} du\right)$$
$$= (x^2)' \frac{\sqrt{1+(x^2)^4}}{x^2} = \frac{2\sqrt{1+x^8}}{x}.$$

So $F''(2) = \sqrt{257}$.

(4) (a) The area of $R$ is

$$\int_1^2 \frac{1}{y} dy = \ln|y|]_1^2 = \ln 2.$$

(b) The volume of $S_1$ is

$$\pi \int_0^{1/2} (2^2 - 1^2)dx + \pi \int_{1/2}^1 \left(\frac{1}{x^2} - 1^2\right) dx$$
$$= \frac{3\pi}{2} + \pi\left(-\frac{1}{x} - x\right)\Big]_{1/2}^1$$
$$= \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi.$$

[1] http://www.math.ucsb.edu/~xichen/math3b00w/mid1sol.pdf

(c) The volume of $S_2$ is

$$\pi \int_1^2 \frac{1}{y^2} dy = \pi \left( -\frac{1}{y} \right) \Bigg]_1^2 = \frac{\pi}{2}.$$

(5) (a) Substitue $u = 1 - 3x$

$$\int \sqrt{1-3x} dx = \frac{1}{-3} \cdot \frac{2}{3}(1-3x)^{3/2} + C = -\frac{2}{9}(1-3x)^{3/2} + C.$$

(b) Substitute $u = \ln x$ $(du = dx/x)$

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{du}{\sqrt{u}} = 2\sqrt{u}\Big]_1^4 = 2.$$

(c)

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \frac{3}{2}(1+2x)^{1/3}\Bigg]_0^{13} = \frac{3}{2}(\sqrt[3]{27} - 1) = 3.$$

(d) Substitue $u = \sqrt{x+1}$, i.e., $x = u^2 - 1$ $(dx = 2udu)$

$$\int_0^3 \frac{x}{\sqrt{1+x}} dx = 2\int_1^2 (u^2 - 1) du = 2\left( \frac{u^3}{3} - u \right)\Bigg]_1^2 = \frac{8}{3}.$$