(1) Take ln on both sides: $\ln e^{ax} = \ln(ce^{bx}) \Rightarrow \ln e^{ax} = \ln c + \ln e^{bx}$ $\Rightarrow ax = \ln c + bx \Rightarrow x = \ln c/(a-b)$.

(2) Solve $y = \ln(\ln x)$ for y: $e^y = e^{\ln(\ln x)} \Rightarrow e^y = \ln x \Rightarrow e^{e^y} = e^{\ln x}$ $\Rightarrow e^{e^y} = x$. So $f^{-1}(x) = e^{e^x}$.

The domain of $f(x)$ is where $x > 0$ and $\ln x > 0$, i.e., $(1, \infty)$. The domain of $f^{-1}(x)$ is $(-\infty, \infty)$. The domain of $f(x)$ is the range of $f^{-1}(x)$ and the domain of $f^{-1}(x)$ is the range of $f(x)$. So the range of $f(x)$ is $(-\infty, \infty)$ and the range of $f^{-1}(x)$ is $(1, \infty)$.

Both $f(x)$ and $f^{-1}(x)$ are elementary functions. So they are continous everywhere on their domains, i.e., $f(x)$ is continuous on $(1, \infty)$ and $f^{-1}(x)$ is continuous on $(-\infty, \infty)$.

(3) Put the curve in the form $y = \sqrt{x^3}$ and calculate its slope at $(1, 1)$:

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{(1+h)^3} - 1}{h} &= \lim_{h\to 0} \frac{(\sqrt{(1+h)^3} - 1)(\sqrt{(1+h)^3} + 1)}{h(\sqrt{(1+h)^3} + 1)} \\
&= \lim_{h\to 0} \frac{(1+h)^3 - 1}{h(\sqrt{(1+h)^3} + 1)} = \lim_{h\to 0} \frac{3h + 3h^2 + h^3}{h(\sqrt{(1+h)^3} + 1)} \\
&= \lim_{h\to 0} \frac{3 + 3h + h^2}{\sqrt{(1+h)^3} + 1} = \frac{3}{2}.
\end{aligned}$$

So the tangent line at $(1, 1)$ is $y - 1 = (3/2)(x - 1)$.

(4) $f(x)$ is obviously continuous everywhere except at $x = 1$. At $x = 1$, $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(cx - 1) = c - 1$ and $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(1 - cx^2) = 1 - c$. For $f(x)$ to be continous at $x = 1$, it is necessary that $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = \lim_{x\to 1} f(x) = f(1)$, i.e., $c - 1 = 1 - c$. So $c = 1$.

(5) Since

$$\begin{aligned}
\lim_{x\to\infty} \frac{\sqrt{4x^2 + 1}}{x + 1} &= \lim_{x\to\infty} \frac{\sqrt{4x^2 + 1}/x}{(x+1)/x} \\
&= \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^2}}\sqrt{4x^2 + 1}}{1 + \frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{\sqrt{4 + \frac{1}{x^2}}}{1 + \frac{1}{x}} = 2
\end{aligned}$$

and

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2+1}}{x+1} = \lim_{x\to-\infty}\frac{\sqrt{4x^2+1}/x}{(x+1)/x}$$
$$= \lim_{x\to-\infty}\frac{-\sqrt{\frac{1}{x^2}}\sqrt{4x^2+1}}{1+\frac{1}{x}}$$
$$= \lim_{x\to-\infty}\frac{-\sqrt{4+\frac{1}{x^2}}}{1+\frac{1}{x}} = -2,$$

the horizontal asymptotes are $y = 2$ and $y = -2$. Notice that $\sqrt{x^2} = |x| = x$ if $x \geq 0$ and $-x$ if $x < 0$.

(6) Since $f(x) = (x^3 - 1)/(x^2 - 1)$ is continous at $x = 0$ (it is an elmentary function and it is defined at $x = 0$), $\lim_{x\to 0} f(x) = f(0) = 1$.

$$\lim_{t\to 9}\frac{9-t}{3-\sqrt{t}} = \lim_{t\to 9}\frac{3^2-(\sqrt{t})^2}{3-\sqrt{t}}$$
$$= \lim_{t\to 9}\frac{(3-\sqrt{t})(3+\sqrt{t})}{3-\sqrt{t}}$$
$$= \lim_{t\to 9}(3+\sqrt{t}) = 6.$$

$$\lim_{x\to\infty}(\sqrt{x+1}-\sqrt{x}) = \lim_{x\to\infty}\frac{(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})}{\sqrt{x+1}+\sqrt{x}}$$
$$= \lim_{x\to\infty}\frac{(x+1)-x}{\sqrt{x+1}+\sqrt{x}}$$
$$= \lim_{x\to\infty}\frac{1}{\sqrt{x+1}+\sqrt{x}}$$
$$= \lim_{x\to\infty}\frac{1/\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})/\sqrt{x}}$$
$$= \lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}}{\sqrt{1+\frac{1}{x}}+1} = 0.$$