

Sample Final

- (1) A particle is moving on a vertical line so that its coordinate at time t is

$$y = e^t - 4e^{t/2} + 3, \quad t \geq 0.$$

- (a) Find the velocity and acceleration.
(b) When is the particle moving upward and when is it moving downward?
- (2) Find the derivative of the following functions, only using the definition of the derivative as a limit.
- (a) $f(x) = 1 - x^2$
(b) $f(x) = 1/\sqrt{x}$.
- (3) Evaluate the following limits. State clearly any rules or theorems that you use. Correct answers, without accompanying work, will receive little or no credit.

(a) $\lim_{x \rightarrow 0} \frac{x^2 + 4}{x^3 + x^2 - 10}$

(b) $\lim_{x \rightarrow \infty} \frac{\cos(e^{x^2} x^1 + 1)}{x^2}$

(c) $\lim_{x \rightarrow \infty} \frac{x^3 - 6x^2 + x}{2x^3 + x - 1}$

(d) $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{x^2}}$

- (4) For each of the following functions, find every point where the function is discontinuous and state why it is discontinuous at each of these points.

(a) $f(x) = \frac{x^2 + e^x}{x^2 - 3x - 4}$

(b)

$$f(x) = \begin{cases} \frac{x^2}{x} & x < 0 \\ x^2 & 0 \leq x < 4 \\ 2^x & 4 \leq x < 5 \\ x & x \geq 5. \end{cases}$$

- (5) Find the derivative of each of the following functions.

(a) $f(x) = e^{x \tan^{-1} x}$

(b) $\ln \left(\frac{x^2 e^x}{\sin x} \right)$

- (c) $(\sin x)^{\sqrt{x}}$
- (d) $\sin(\tan(x^2))$

(6) Find local and absolute extremum values of the function $f(x) = xe^{-x}$ on the interval $[0, 2]$.

(7) Find the equations of the tangent line to:

- (a) $f(x) = \sqrt{x}$ at $x = 1$
- (b) $y = x^2(5 - x)^{1/2}$ at the point $(1, 2)$

(8) If 600 cm^2 of material is available to make a box with a square base and an open top. Find the largest possible volume of the box.

(9) Sketch the graphs of each of the following functions.

- (a) $f(x) = x^3 - 9x$
- (b)

$$f(x) = \frac{x^2}{x + 1}$$

- (c) $f(x) = \sqrt{x + 1} - \sqrt{x}$

(10) Mold is growing in my refrigerator. The amount of mold, in grammes, is $f(t)$, where t is the time in hours after noon, on Monday May 8th. Your explanations of the meaning should be written in plain English and not involve any mathematical symbols.

- (a) What does $f(3) = 10$ mean?
- (b) What does $f'(5) = 20$ mean?
- (c) What does $f^{-1}(100) = 24$ mean?

(11) Use the Intermediate Value Theorem to show (explain carefully how you use Intermediate Value Theorem):

- (a) $f(x) = x^4 + x^2 + x - 2$ has a root in the interval $[-1, 1]$.
- (b) $\cos(\pi x) = x$ has a solution.

(12) The volume of a cube is increasing at rate of $30 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 10 cm ?