1. Let x be the distance (in ft) from the bottom of the ladder to the wall. Then $x = 10\cos\theta$. Hence

$$\frac{dx}{dt} = -(10\sin\theta)\frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = -(10\sin\theta)^{-1}\left(\frac{dx}{dt}\right).$$

Since $dx/dt = 1$, $d\theta/dt = -(10\sin(\pi/3))^{-1} = -\sqrt{3}/15\ \mathrm{s}^{-1}$ when $\theta = \pi/3$.

2. Use implicit differentiation:

$$\begin{aligned}
&\frac{d}{dx}(x^3y + 2xy^3) = 0\\
&\Rightarrow 3x^2y + x^3\frac{dy}{dx} + 2y^3 + 6xy^2\frac{dy}{dx} = 0\\
&\Rightarrow (x^3 + 6xy^2)\frac{dy}{dx} + (3x^2y + 2y^3) = 0\\
&\Rightarrow \frac{dy}{dx} = -\frac{3x^2y + 2y^3}{x^3 + 6xy^2}\\
&\Rightarrow \left.\frac{dy}{dx}\right|_{(1,1)} = -\frac{5}{7}.
\end{aligned}$$

So the tangent line is $5(x-1) + 7(y-1) = 0$.

3.

$$\begin{aligned}
f'(x) &= (2^{3^{\cos x}})' = (2^{3^{\cos x}})(\ln 2)(3^{\cos x})'\\
&= -(2^{3^{\cos x}})(\ln 2)(3^{\cos x})(\ln 3)(\sin x).
\end{aligned}$$

4. 4. Let x be the height of the water (in m) and r be the rate of the water being pumped into the tank (in $\mathrm{m}^3/\mathrm{min}$). Then the volume of the water is

$$V = \frac{1}{12}\pi\left(\frac{4x}{6}\right)^2 x = \frac{1}{27}\pi x^3.$$

So

$$\frac{dV}{dt} = \frac{1}{9}\pi x^2\frac{dx}{dt}$$

On the other hand

$$\frac{dV}{dt} = r - 0.01 \text{ and } \frac{dx}{dt} = 0.2.$$

So when $x = 2$,

$$r - 0.01 = \frac{0.8\pi}{9}.$$

So $r = 0.289\ \mathrm{m}^3/\mathrm{min}$.

5. Since $f(g(x)) = x$, $f'(g(x))g'(x) = 1$. So

$$g'(e+1) = \frac{1}{f'(g(e+1))}.$$

Since $g(e+1) = 1$, $g'(e+1) = 1/f'(1) = 1/(e+1)$.

6. Use logarithmic differentiation:

$$\begin{aligned}
\ln f(x) &= \frac{1}{2}\ln(\ln x+1) - \frac{1}{2}\ln(\ln x-1) \\
\Rightarrow \frac{f'(x)}{f(x)} &= \frac{1}{2}\frac{1}{x(\ln x+1)} - \frac{1}{2}\frac{1}{x(\ln x-1)} \\
\Rightarrow \frac{f'(x)}{f(x)} &= -\frac{1}{x(\ln x+1)(\ln x-1)} \\
\Rightarrow f'(x) &= -\frac{1}{x}\sqrt{\frac{1}{(\ln x+1)(\ln x-1)^3}}
\end{aligned}$$

7. Since $g'(x) = \sec x \tan x$, $g''(x) = \sec^3 x + \sec x \tan^2 x$ and $g'''(x) = 5\sec^3 x \tan x + \sec x \tan^3 x$, $g'''(\pi/4) = 11\sqrt{2}$.
8. Use logarithmic differentiation:

$$\begin{aligned}
\ln f(x) &= x\ln(\cos^{-1} x) \\
\Rightarrow \frac{f'(x)}{f(x)} &= \ln(\cos^{-1} x) - \frac{x}{\sqrt{1-x^2}(\cos^{-1} x)} \\
\Rightarrow f'(x) &= (\cos^{-1} x)^x\left(\ln(\cos^{-1} x) - \frac{x}{\sqrt{1-x^2}(\cos^{-1} x)}\right)
\end{aligned}$$

9. Let S be the surface area and x be the diameter. Then $S = \pi x^2$ and $dS/dt = 2\pi x(dx/dt)$. So $dx/dt = -1/(20\pi)$. So the diameter decreases at the rate of $1/(20\pi)$ cm/min.
10. The graph of f has a horizontal tangent at $f'(x) = 1 - \cos x = 0$, i.e., $\cos x = 1$. So it has a horizontal tangent at $x = 2k\pi$ for k any integers.