

Solution for Practice Midterm I¹

1. Suppose that the entire flight takes x hours and let t be the time variable (in hours). In the first half hour, the plane flies $200(0.5) = 100$ miles. From $t = 0.5$ to $t = x - 2$, the plane flies $1950 - 850 - 100 = 1000$ miles. So the speed of the plane from $t = 0.5$ to $t = x - 0.5$ is $1000/(x - 2.5)$. Hence the plane flies $(x - 1)(1000/(x - 2.5))$ miles from $t = 0.5$ to $t = x - 0.5$. Therefore,

$$(x - 1)\frac{1000}{x - 2.5} + 200(0.5 + 0.5) = 1950.$$

Solve the equation for x and we obtain $x = 4.5$.

2. The volume of the swimming pool is $5 \cdot 50 \cdot 2 = 500 \text{ m}^3$. Suppose that $y \text{ m}^3$ of water should be drained. After $y \text{ m}^3$ of water is drained, the remaining $(500 - y) \text{ m}^3$ of water contains $(500 - y)x$ grams of chlorine. The $y \text{ m}^3$ of replacement contains $3y$ grams of chlorine. So the pool ends up with $(500 - y)x + 3y$ grams of chlorine. Therefore,

$$(500 - y)x + 3y = 500(0.02).$$

Solve it for y and we obtain $y = (10 - 500x)/(3 - x)$.

3. (a)

$$\begin{aligned} \int_0^a (ax - 3)^2 dx &= \int_0^a (a^2 x^2 - 6ax + 9) dx \\ &= \left[\frac{1}{3} a^2 x^3 - 3ax^2 + 9x \right]_0^a = \frac{1}{3} a^5 - 3a^3 + 9a \end{aligned}$$

- (b)

$$\int_0^4 \sqrt{x} dx = \left[\frac{2}{3} x^{3/2} \right]_0^4 = \frac{16}{3}$$

- (c)

$$\int \frac{2 + x^2}{x^2} dx = \int \frac{2}{x^2} dx + \int 1 dx = -\frac{2}{x} + x + C$$

¹<http://www.math.ucsb.edu/~xichen/math34b02w/p1sol.pdf>

4. (a) After t minutes, the total amount of water removed from the tank is $\int_0^t (1+x)dx = t + t^2/2$ gallons. So there are $16 - t - t^2/2$ gallons remaining after t minutes.
- (b) Suppose that the tank is half empty after t minutes. Then

$$t + \frac{t^2}{2} = 8.$$

Solve it and we obtain $t = \sqrt{17} - 1$.

5. (a) Let $v(t)$ be the velocity of the car. Then the distance travelled by the car in 4 hours is

$$\int_0^4 v(t)dt = \int_0^2 v(t)dt + \int_2^4 v(t)dt = \frac{1}{2}(60)(2) + (60)(2) = 180 \text{ mile.}$$

- (b) The average velocity is $180/4 = 45$ mph.
- (c) The acceleration is $60/2 = 30$ mile/hour² after one hour.