Solution for Midterm I [1]

1. Let $x$ be the speed of the first plane (in mph). Then the speed of the second plane is $x + 50$. It takes $500/x$ hours for the first plane to fly the first 500 miles and $1000/(x + 50)$ for the second plane to fly the second 1000 miles. Therefore,

$$\frac{500}{x} + \frac{1000}{x+50} = \frac{13}{2}.$$

Solve the equation by multiplying both sides by $x(x + 50)$:

$$500(x + 50) + 1000x = \frac{13}{2}x(x + 50).$$

Further simplify it and we obtain

$$13x^2 - 2350x - 50000 = 0.$$

Solve it and we have $x = 200$ or $-250/13$. Of course, $x$ must be positive. Therefore, $x = 200$.

2. Suppose that it takes $y$ hours for the concentration to reach $x$ mg/l. After $y$ hours, the tank has $1000 + 20y$ liters of water and $(4)(20y) = 80y$ mg of detergent. So

$$80y = (1000 + 20y)x.$$

Solve it for $y$ and we obtain $y = 1000x/(80 - 20x) = 50x/(4 - x)$.

3. (a)

$$\begin{aligned}\int \frac{(1+x)^2}{x}dx &= \int \frac{1+2x+x^2}{x}dx \\ &= \int \frac{1}{x}dx + \int 2dx + \int xdx \\ &= \ln|x| + 2x + \frac{x^2}{2} + C.\end{aligned}$$

[1] http://www.math.ucsb.edu/~xichen/math34b02w/mid1sol.pdf

(b)

$$\begin{aligned}\int_1^2 (1+2x)(2+x)dx &= \int_1^2 (2+5x+2x^2)dx \\ &= \left[2x+\frac{5}{2}x^2+\frac{2}{3}x^3\right]_1^2 \\ &= 2+\frac{15}{2}+\frac{14}{3}=\frac{85}{6}.\end{aligned}$$

(c)

$$\begin{aligned}\int_0^4 x(x+\sqrt{x})dx &= \int_0^4 (x^2+x^{3/2})dx \\ &= \left[\frac{x^3}{3}+\frac{2}{5}x^{5/2}\right]_0^4 \\ &= \frac{64}{3}+\frac{64}{5}=\frac{512}{15}.\end{aligned}$$

4. (a) The amount of water pumped into the tank is $\int_0^t (1+2x)dx = t+t^2$ gallons after $t$ minutes. So there is $40+t+t^2$ gallons of water in the tank after $t$ minutes.

   (b) Solve the equation $40+t+t^2=60$ and we obtain $t=4$.

5. (a) The distance travelled by the car in 7 hours is

$$\int_0^7 v(t)dt = (1/2)(2\cdot 60)+2\cdot 60+(1/2)(3\cdot 60)=270 \text{ mi}.$$

   (b) The distance travelled by the car in the first 6 hours is

$$\int_0^6 v(t)dt = 270-(1/2)(60/3)\cdot 1 = 260 \text{ mi}.$$

   So the average velocity is $260/6 = 130/3$ mph.

   (c) The acceleration of the car after 3 hours is 0 mi/h$^2$. And the acceleration of the car after 5 hours is $-60/3=-20$ mi/h$^2$.