9.3.4  (a) \(\int e^{-3x}dx = -e^{3x}/3 + C\)

(b) Write \(2^x = (e^{\ln 2})^x = e^{(\ln 2)x}\). So \(\int 2^x dx = e^{(\ln 2)x}/(\ln 2) + C = 2^x/(\ln 2) + C\).

(c) Write \(10^{x+3} = 10^3 \cdot e^{(\ln 10)x}\). So \(\int 10^{x+3} dx = 10^3 e^{(\ln 10)x}/(\ln 10) + C = 10^{x+3}/(\ln 10) + C\).

9.6.3  \(\int_0^{10} 2\pi x dx = 100\pi\).

9.6.8  (a) \(\int_0^4 f(x)dx = 2 \cdot 4 = 8\) and \(\int_4^6 f(x)dx = (2 \cdot 2)/2 = 2\). Therefore, \(\int_0^6 f(x)dx = 8 + 2\) gallons of water left the tank after 6 hours.

(b) Let \(t\) be the hours it takes for 9.5 gallons of water to leave the tank. Since 9.5 > 8, \(t\) is between 4 and 6. \(\int_t^6 f(x)dx = \int_0^6 f(x)dx - \int_0^t f(x)dx = 10 - 9.5 = 0.5\). On the other hand, \(\int_t^6 f(x)dx\) is the area of a right triangle with base \(6 - t\) and height \(6 - t\). So \((6 - t)^2/2 = 0.5\). Solve it and we obtain \(t = 5\) or \(t = 7\). Obviously, \(t = 7 > 6\) is impossible. So \(t = 5\).

10.2.53  Let \(h\) be the height of the box, \(w\) be the width of the base and \(l\) be the length of the base (all in m). Then \(w = 2h\) and \(lwh = 5\). And hence \(l = 5/wh = 5/(2h^2)\). The total surface area is

\[2wh + 2lh + 2l = 2(2h)h + 2(5/(2h^2))h + 2(5/(2h^2))(2h) = 4h^2 + 15/h.\]

10.2.54  Suppose that the water level is \(x\) cm above the bottom of the tank. The volume of water is \(V = \pi(200)^2x = 4(10)^4\pi x\) cm\(^3\), i.e., \(V = 40\pi x\) liters. Since \(dV/dt = 40\pi dx/dt = -5\), \(dx/dt = -1/(8\pi)\) cm per minute, i.e., \(-60/(8\pi) = -15/(2\pi)\) cm per hour. So the water level is falling \(15/(2\pi)\) cm per hour.

Page 187

1http://www.math.ucsb.edu/~xichen/math34b02w/hw34key.pdf
11.0.10 After transferring 2 liters from A to B, B contains $2 + 2 = 4$ liters of solution with concentration:
\[
\frac{2 \cdot 0.05 + 2 \cdot 0.1}{2 + 2} = 0.075.
\]
Then after transferring 1 liter from B to C, C contains $1 + 4 = 5$ liters of solution with concentration:
\[
\frac{1 \cdot 0.075 + 4 \cdot 0}{1 + 4} = 0.015.
\]
Finally after transferring 2 liters from C to A, A contains $2 + (3 - 2) = 3$ liters of solution with concentration:
\[
\frac{2 \cdot 0.015 + (3 - 2) \cdot 0.05}{2 + (3 - 2)} = (8/3)\%.
\]

11.0.15 (a) Suppose it takes $t$ hours for the concentration to reach 2 mg/l. After $t$ hours, the volume of the tank is $1000 + 20t$ liters and the amount of detergent is $4(20)t = 80t$ mg. So
\[
\frac{80t}{1000 + 20t} = 2.
\]
Solve it and we obtain $t = 50$.

(b) Same idea as (a). Let $t$ be the hours it takes for the concentration to reach $x$ mg/l. Then
\[
\frac{80t}{1000 + 20t} = x.
\]
Solve it for $t$ and we obtain $t = 50x/(4 - x)$.

(d) If $x = 5$, $t$ is negative. It means it is impossible for the concentration to reach 5 mg/l.

11.0.55 The whole process can be described by the following table.
Initially | A’s Money | B’s Money | C’s Money
---|---|---|---
A gives half to B | \( \frac{a}{2} \) | \( b + \frac{a}{2} \) | \( c \)
A takes half from C | \( \frac{a}{2} + \frac{c}{2} \) | \( b + \frac{a}{2} \) | \( \frac{c}{2} \)
B gives half to C | \( \frac{a}{2} + \frac{c}{2} \) | \( b + \frac{a}{2} + \frac{c}{2} \) | \( \frac{c}{2} + \frac{a}{4} \)
B takes half from A | \( \frac{a}{2} + \frac{b}{2} \) | \( b + \frac{a}{2} + \frac{c}{2} \) | \( \frac{c}{2} + \frac{b}{2} + \frac{a}{4} \)
C gives half to A | \( \frac{3}{8}a + \frac{b}{2} + \frac{c}{4} \) | \( \frac{b}{2} + \frac{a}{2} + \frac{c}{2} \) | \( \frac{3}{8}a + \frac{b}{2} + \frac{3}{8}c \)
C takes half from B | \( \frac{3}{8}a + \frac{b}{2} + \frac{c}{4} \) | \( b + \frac{a}{2} + \frac{c}{2} \) | \( \frac{3}{8}a + \frac{b}{2} + \frac{3}{8}c \)

Therefore, in the end, C has \( \left( \frac{3}{8}c + \frac{b}{2} + \frac{3}{8}a \right) - \left( \frac{3}{8}a + \frac{b}{2} + \frac{b}{4} \right) = \frac{b}{4} - \frac{c}{8} \) dollars more than A.

11.0.59 Let \( x \) be the length of the edge of the base and \( y \) be the height of the box. Then the volume of the box is \( x^2y \) and hence \( x^2y = 20 \). Consequently, \( y = 20/x^2 \). The top and bottom of box both have area \( x^2 \). So the total cost of the top and bottom is \( 2x^2 \cdot 30 = 60x^2 \). Each of the four side faces has area \( xy = x(20/x^2) = 20/x \). So the total cost of the sides is \( 4(20/x) \cdot 20 = 1600/x \). Therefore, the total cost of the box is \( 60x^2 + 1600/x \).

11.0.60 Suppose that they meet each other at \( x \) miles from LA. The speed of plane A is \( 500 - 100 = 400 \) mph. When the two meet, plane A has flied \( (5000 - x)/400 \) hours. The speed of plane B is \( 500 + 100 = 600 \) mph. When the two meet, plane B has flied \( x/600 \) hours. Therefore,

\[
\frac{5000 - x}{400} - \frac{x}{600} = 1.
\]

Solve it and we have \( x = 2760 \). So they meet 2760 miles off LA.

11.0.62 After half of the contents of B is poured into A, A contains \( 1+1 = 2 \) liters of mixture with \( 1 \cdot 0.1 + 1 \cdot 0.5 = 0.6 \) liters of oil. Suppose that \( x \) liters of oil needed to added to A to produce a mixture which is 60% of oil. After \( x \) liters of oil is added to A, A contains \( 2 + x \) liters of mixture with \( 0.6 + x \) liters of oil. Therefore,

\[0.6 + x = 0.6(2 + x).
\]

Solve it and we obtain \( x = 1.5 \).