Solution for Selected Homework Problems (Week 3-4) [1]

Page 165

9.3.4 (a) $\int e^{-3x}dx = -e^{3x}/3 + C$

(b) Write $2^x = (e^{\ln 2})^x = e^{(\ln 2)x}$. So $\int 2^x dx = e^{(\ln 2)x}/(\ln 2) + C = 2^x/(\ln 2) + C$.

(c) Write $10^{x+3} = 10^3 \cdot e^{(\ln 10)x}$. So $\int 10^{x+3}dx = 10^3 e^{(\ln 10)x}/(\ln 10) + C = 10^{x+3}/(\ln 10) + C$.

Page 173

9.6.3 $\int_0^{10} 2\pi x dx = 100\pi$.

9.6.8 (a) $\int_0^4 f(x)dx = 2 \cdot 4 = 8$ and $\int_4^6 f(x)dx = (2 \cdot 2)/2 = 2$. Therefore, $\int_0^6 f(x)dx = 8 + 2$ gallons of water left the tank after 6 hours.

(b) Let t be the hours it takes for 9.5 gallons of water to leave the tank. Since $9.5 > 8$, t is between 4 and 6. $\int_t^6 f(x)dx = \int_0^6 f(x)dx - \int_0^t f(x)dx = 10 - 9.5 = 0.5$. On the other hand, $\int_t^6 f(x)dx$ is the area of a right triangle with base $6 - t$ and height $6 - t$. So $(6 - t)^2/2 = 0.5$. Solve it and we obtain $t = 5$ or $t = 7$. Obviously, $t = 7 > 6$ is impossible. So $t = 5$.

Page 184

10.2.53 Let h be the height of the box, w be the width of the base and l be the length of the base (all in m). Then $w = 2h$ and $lwh = 5$. And hence $l = 5/wh = 5/(2h^2)$. The total surface area is

$$2wh + 2lh + 2lw = 2(2h)h + 2(5/(2h^2))h + 2(5/(2h^2))(2h) = 4h^2 + 15/h.$$

10.2.54 Suppose that the water level is x cm above the bottom of the tank. The volume of water is $V = \pi(200)^2 x = 4(10)^4 \pi x$ cm^3, i.e., $V = 40\pi x$ liters. Since $dV/dt = 40\pi dx/dt = -5$, $dx/dt = -1/(8\pi)$ cm per minute, i.e., $-60/(8\pi) = -15/(2\pi)$ cm per hour. So the water level is falling $15/(2\pi)$ cm per hour.

Page 187

[1] http://www.math.ucsb.edu/~xichen/math34b02w/hw34key.pdf

11.0.10 After transferring 2 liters from A to B, B contains $2+2=4$ liters of solution with concentration:

$$\frac{2\cdot 0.05+2\cdot 0.1}{2+2}=0.075.$$

Then after transferring 1 liter from B to C, C contains $1+4=5$ liters of solution with concentration:

$$\frac{1\cdot 0.075+4\cdot 0}{1+4}=0.015.$$

Finally after transferring 2 liters from C to A, A contains $2+(3-2)=3$ liters of solution with concentration:

$$\frac{2\cdot 0.015+(3-2)\cdot 0.05}{2+(3-2)}=(8/3)\%.$$

11.0.15 (a) Suppose it takes t hours for the centration to reach 2 mg/l. After t hours, the volume of the tank is $1000+20t$ liters and the amount of detergent is $4(20)t=80t$ mg. So

$$\frac{80t}{1000+20t}=2.$$

Solve it and we obtain $t=50$.

(b) Same idea as (a). Let t be the hours it takes for the centration to reach x mg/l. Then

$$\frac{80t}{1000+20t}=x.$$

Solve it for t and we obtain $t=50x/(4-x)$.

(d) If $x=5$, t is negative. It means it is impossible for the centration to reach 5 mg/l.

11.0.55 The whole process can be described by the following table.

	A's Money	B's Money	C's Money
Initially	a	b	c
A gives half to B	$\frac{a}{2}$	$b+\frac{a}{2}$	c
A takes half from C	$\frac{a}{2}+\frac{c}{2}$	$b+\frac{a}{2}$	$\frac{c}{2}$
B gives half to C	$\frac{a}{2}+\frac{c}{2}$	$\frac{b}{2}+\frac{a}{4}$	$\frac{c}{2}+\frac{b}{2}+\frac{a}{4}$
B takes half from A	$\frac{a}{4}+\frac{c}{4}$	$\frac{b}{2}+\frac{a}{2}+\frac{c}{4}$	$\frac{c}{2}+\frac{b}{2}+\frac{a}{4}$
C gives half to A	$\frac{3}{8}a+\frac{c}{2}+\frac{b}{4}$	$\frac{b}{2}+\frac{a}{2}+\frac{c}{4}$	$\frac{c}{4}+\frac{b}{4}+\frac{a}{8}$
C takes half from B	$\frac{3}{8}a+\frac{c}{2}+\frac{b}{4}$	$\frac{b}{4}+\frac{a}{4}+\frac{c}{8}$	$\frac{3}{8}c+\frac{b}{2}+\frac{3}{8}a$

Therefore, in the end, C has $(\frac{3}{8}c+\frac{b}{2}+\frac{3}{8}a)-(\frac{3}{8}a+\frac{c}{2}+\frac{b}{4})=\frac{b}{4}-\frac{c}{8}$ dollars more than A.

11.0.59 Let x be the length of the edge of the base and y be the height of the box. Then the volume of the box is x^2y and hence $x^2y=20$. Consequently, $y=20/x^2$. The top and bottom of box both have area x^2. So the total cost of the top and bottom is $2x^2\cdot 30=60x^2$. Each of the four side faces has area $xy=x(20/x^2)=20/x$. So the total cost of the sides is $4(20/x)\cdot 20=1600/x$. Therefore, the total cost of the box is $60x^2+1600/x$.

11.0.60 Suppose that they meet each other at x miles from LA. The speed of plane A is $500-100=400$ mph. When the two meet, plane A has flied $(5000-x)/400$ hours. The speed of plane B is $500+100=600$ mph. When the two meet, plane B has flied $x/600$ hours. Therefore,

$$\frac{5000-x}{400}-\frac{x}{600}=1.$$

Solve it and we have $x=2760$. So they meet 2760 miles off LA.

11.0.62 After half of the contents of B is poured into A, A contains $1+1=2$ liters of mixture with $1\cdot 0.1+1\cdot 0.5=0.6$ liters of oil. Suppose that x liters of oil needed to added to A to produce a mixture which is 60% of oil. After x liters of oil is added to A, A contains $2+x$ liters of mixture with $0.6+x$ liters of oil. Therefore,

$$0.6+x=0.6(2+x).$$

Solve it and we obtain $x=1.5$.