

Solution for Selected Homework Problems (Week 1-2) <sup>1</sup>

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**9.1.4**  $\int_0^1 f(x)dx$  is the area of a right triangle with base 1 and height 2 and hence  $\int_0^1 f(x)dx = (1 \cdot 2)/2 = 1$ .  $\int_1^3 f(x)dx$  is the area of a rectangle with length 2 and width 2 and hence  $\int_1^3 f(x)dx = 2 \cdot 2 = 4$ .  $\int_3^5 f(x)dx$  is the area of a right triangle with base 2 and height 2 and hence  $\int_3^5 f(x)dx = (2 \cdot 2)/2$ . So the total area of the region is  $\int_0^7 f(x)dx = 1 + 4 + 2 = 7$ . Since  $1 + 4 = 5 < 6$ ,  $t$  must be between 3 and 5. We have

$$\int_t^5 f(x)dx = \int_0^5 f(x)dx - \int_0^t f(x)dx = 7 - 6 = 1.$$

On the other hand,  $\int_t^5 f(x)dx$  is the area of a right triangle with base  $5 - t$  and height  $5 - t$  and hence  $\int_t^5 f(x)dx = (5 - t)^2/2$ . Solve the equation  $(5 - t)^2/2 = 1$  and we have  $t = 5 \pm \sqrt{2}$ . Since  $5 + \sqrt{2} > 5$ , which is impossible,  $t = 5 - \sqrt{2}$ .

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**11.0.1** Let  $x$  be the length and  $y$  be the width (in cm). Then  $2(x + y) = 24$  is the perimeter and  $xy = 32$  is the area. By  $2(x + y) = 24$ , we may express  $x$  in terms of  $y$ :  $y = 12 - x$ . Plugging it into  $xy = 32$ , we obtain  $x(12 - x) = 32 \Rightarrow x^2 - 12x + 32$ . This is a quadratic equation and the solution is  $x = 4$  or  $8$ . Hence  $y = 8$  or  $4$ . Therefore, the length and width are 4 and 8 cm or 8 and 4 cm.

**11.0.2** Let  $x$  be the speed of the first plane (in mph). Then the speed of the second plane is  $x + 50$ . It takes  $500/x$  hours for the first plane to fly the first 500 miles and  $1000/(x + 50)$  for the second plane to fly the second 1000 miles. Therefore,

$$\frac{500}{x} + \frac{1000}{x + 50} = \frac{13}{2}.$$

Solve the equation by multiplying both sides by  $x(x + 50)$ :

$$500(x + 50) + 1000x = \frac{13}{2}x(x + 50).$$

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<sup>1</sup><http://www.math.ucsb.edu/~xichen/math34b02w/hw12key.pdf>

Further simplify it and we obtain

$$13x^2 - 2350x - 50000 = 0.$$

Solve it and we have  $x = 200$  or  $-250/13$ . Of course,  $x$  must be positive. Therefore,  $x = 200$ .

**11.0.3** Following the hint, let  $x$  be the time taken by George (in hours) to mow the lawn. Then it takes Henry  $x - 1$  hours to mow the lawn. Since it takes  $x$  hours for George to mow the lawn, he mow  $1/x$  of the lawn per hour. Similarly, Henry mow  $1/(x - 1)$  of the lawn per hour. Together, they mow  $1/x + 1/(x - 1)$  of the lawn per hour. Therefore,

$$\frac{1}{x} + \frac{1}{x - 1} = \frac{1}{1.2}$$

since it takes them together 1.2 hours (1 hour and 12 mininutes) to mow the lawn. Solve the above equation by multiplying  $x(x - 1)$  on both sides:

$$(x - 1) + x = \frac{1}{1.2}x(x - 1).$$

Simplify it and we obtain

$$x^2 - 3.4x + 1.2 = 0.$$

The solutions are  $x = 3$  and  $x = 2/5$ . If  $x = 2/5$ ,  $x - 1 < 0$ , which is impossible. Therefore,  $x = 3$ . It takes Henry  $x - 1 = 2$  hours to mow the lawn.

**11.0.4** Let  $x$  be the smaller integer. Then the large integer is  $x + 2$ . We have

$$\frac{1}{x} + \frac{1}{x + 2} = \frac{8}{15}.$$

Multiply  $x(x + 2)$  on both sides:

$$(x + 2) + x = \frac{8}{15}x(x + 2).$$

Simplify it and we obtain

$$4x^2 - 7x - 15 = 0.$$

The solutions are  $x = 3$  and  $x = -5/4$ . Of course,  $-5/4$  is not an integer. So  $x = 3$ . The two integers are 3 and 5.

**11.0.5** The distance between A and C is  $|2 - (-3)| = 5$ . The distance between B and D is  $|x - 13|$ . Therefore,

$$|x - 13| = 5.$$

Hence  $x - 13 = 5$  or  $x - 13 = -5 \Rightarrow x = 18$  or  $x = 8$ .

**11.0.6** Follow the hint by letting  $x$  be the amount of the pool filled by the small pipe in one hour. Since it takes small pipe twice the time that the large pipe takes to fill the pool, the amount of the pool filled by the large pipe in one hour is  $2x$ . Therefore,

$$x + 2x = \frac{1}{7/3}.$$

So  $x = 1/7$ . So it takes the large pipe  $1/(2x) = 7/2$  hours to fill the pool.

**11.0.7** Let  $x$  be the speed of car A (in mph). Then car B has speed  $x - 10$  mph. It takes car A  $300/x$  hours and car B  $300/(x - 10)$  hours to travel 300 miles, respectively. So

$$\frac{300}{x - 10} - \frac{300}{x} = 1.$$

Multiply both sides by  $x(x - 10)$ :

$$300x - 300(x - 10) = x(x - 10).$$

Simplify it and we obtain

$$x^2 - 10x - 3000 = 0.$$

The solutions are  $x = 60$  and  $x = -50$ . Of course,  $x = -50$  is impossible. So  $x = 60$  and  $x - 10 = 50$ . The average speed is  $(60 + 50)/2 = 55$  mph.

**11.0.8** After 3 hours, task A is half done and task B is started. After another 2 hours, 5 hours have been spent on task A and 2 hours have been spent on task B so the total hours spent on A and B are 7 and task C is started. After another 2 hours, C is about to end in one hour and task D is started. After another 2 hours, D is finished. So D ends after  $3 + 2 + 2 + 2 = 9$  hours.

**11.0.9** Since the ratio of W, X and Y is  $2 : 1 : 3$  in Z, there are  $20(2/6) = 20/3$  grams of W,  $20(1/6) = 10/3$  grams of X and  $20(3/6) = 10$  grams of Y in 20 grams of Z. The ratio of W and X in Y is  $2 : 1$ . So there are  $10(2/3) = 20/3$  grams of W and  $10(1/3) = 10/3$  grams of X in 10 grams of Y. Therefore, there are  $20/3 + 20/3 = 40/3$  grams of W and  $10/3 + 10/3 = 20/3$  grams of X in 20 grams of Z.