

PRINT NAME: _____

PERM NUMBER: _____

1. No books and notes are allowed.
2. You may use a calculator and a notecard.
3. Show your work in details.

Problem	Score	Problem	Score
1 (15)		2 (20)	
3 (20)		4 (20)	
5 (20)		6 (30)	
7 (15)		8 (30)	
9 (30)			
Total (200)			

1. (15 points) The following is the graph of a sine wave function

$$f(x) = A \sin(kx) + C,$$

where A, k, C are constants and $A > 0$. Find (a) the value of A, k, C
(b) the period, frequency and amplitude of $f(x)$.

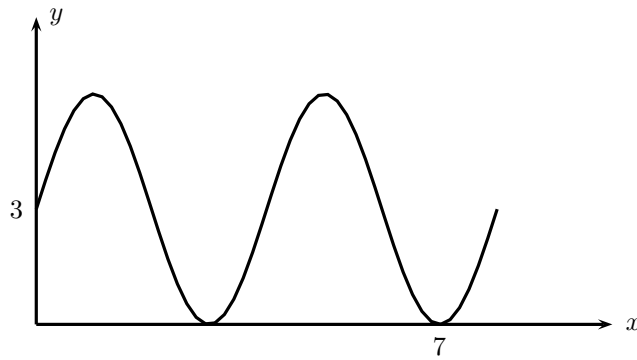


Figure 1: Problem 1

2. (20 points) Find the local maxima and minima of the function

$$f(x) = x^4 + 4x^3 + 4x^2.$$

Clearly show how you apply the second derivative test.

3. (20 points) Evaluate the following integrals

(a) $\int (x^3 + x + 1)dx$

(b) $\int_1^2 \frac{1+x^2}{x} dx$

(c) $\int \frac{1+e^{2x}}{e^x} dx$

(d) $\int_0^\pi (\sin(x/4) - \cos(x/4))dx$

4. (20 points) A lake has a volume of 10^8 m^3 . A river flows into the lake at the rate of $2 \times 10^5 \text{ m}^3/\text{day}$. The same amount of pure water evaporates from the lake each day, so the volume of the lake does not change. The river contains 5% pollution. The lake is initially unpolluted.

(a) (10 points) What is the percentage pollution in the lake after t days?

(b) (10 points) When does the pollution in the lake reach 1%?

5. (20 points) Find the derivatives of the following functions.

(a) $e^{2x} \sin(2x)$

(b) $(\sin x)^2$

(c) $\frac{1+x}{x^2}$

(d) $(x^2 + x) \cos(2x)$

6. (30 points) Let $f(x, y) = x^2 + xy + y^2 + x + y$.

(a) (10 points) Find $\partial f/\partial x$ and $\partial f/\partial y$.

(b) (5 points) Find the tangent plane of $z = f(x, y)$ at the point $(x, y) = (2, 1)$.

(c) (5 points) Find the linear approximation (tangent plane approximation) of $f(x, y)$ at $(2, 1)$.

(d) (10 points) Find the minimum of $f(x, y)$.

7. (15 points) Plane A flies at a constant speed from New York to Los Angeles along a route which is 2000 miles. Plane B flies in the opposite direction at a constant speed which is 100 mph faster than plane A. Plane B takes off one hour after plane A. They land at the same moment. How far are they from Los Angeles when they pass?

8. (30 points) A certain sample of isotope decays radioactively. The following graph shows its mass m (in grams) as a function of time t (in years).

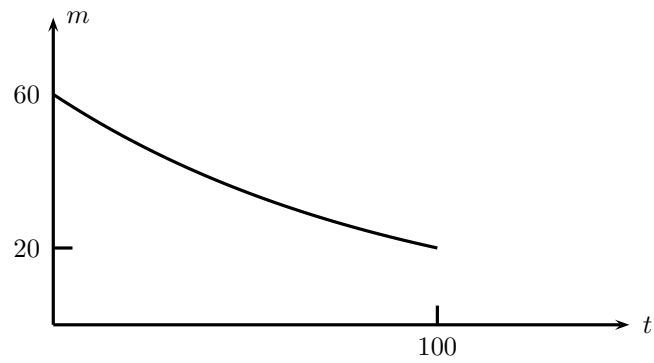


Figure 2: Problem 8

- (a) (10 points) Find the constant k such that $m(t)$ satisfies the differential equation $m' = -km$.

(b) (10 points) What is the mass after 50 years?

(c) (10 points) When will the mass be 10 grams? (You may leave $\ln(\dots)$ in your answer)

9. (30 points) Find the solutions of the logistic equation $y' = y(3 - 2y)$ with initial condition $y(0) = 1$ and another with $y(1) = 1$.