A Sample Final[1]

1. (10 points) Find the frequency, period and amplitude of the wave function $10\sin(10t + 10) + 10$.

2. (20 points) Find the local maxima and minima of the function $f(x) = x^4 - x^2$. Clearly show how you apply the second derivative test.

3. (20 points) Evaluate the following integrals

   (a) $\int x(x+1)dx$

   (b) $\int_1^4 (1+\sqrt{x})/x dx$

   (c) $\int e^x(1+e^{-x})dx$

   (d) $\int_0^\pi (\sin(x/2) + \cos(x/2))dx$

4. (20 points) Beaker A contains 1 liter which is 10% oil and the rest is vinegar, thoroughly mixed up. Beaker B contains 2 liters which is 50% oil and 50% vinegar, completely mixed up. Half of the contents of B are poured into A, then completely mixed up. How much oil should now be added to A to produce a mixture which is 60% oil?

5. (20 points) Find the derivatives of the following functions.

   (a) xe^{2x}

   (b) $\sin(4x)\cos(2x)$

   (c) $e^x\cos(2x)$

   (d) $(x + 1/x)\sin x$

6. (30 points) Let $f(x, y) = 2x^2 + xy + 2y^2 + 5y + 7$.

   (a) (5 points) Find $\partial f/\partial x$ and $\partial f/\partial y$.

   (b) (10 points) Find the tangent plane of $z = f(x, y)$ at the point $(x, y) = (2, 1)$.

   (c) (5 points) Find the linear approximation (tangent plane approximation) of $f(x, y)$ at $(2, 1)$.

   (d) (10 points) Find the minimum of $f(x, y)$.

[1] http://www.math.ucsb.edu/~xichen/math34b02w/f.pdf

7. (20 points) A car travels 365 miles in 6 hours. It travels at 50 mph for the first two hours and travels the last 165 miles at 60 mph. It travels at constant speed for the remaining time.

   (a) (10 points) How long did it take to travel the first 150 miles?

   (b) (10 points) Where was the car after 3 hours?

8. (30 points) The mass of a colony of bacteria is growing at a rate proportional to its mass. The following graph shows the mass $y(t)$ as a function of time t with y in grams and t in minutes.

   (a) (10 points) Find the constant k such that $y(t)$ satisfies the differential equation $y' = ky$.

   (b) (10 points) What is the mass after 10 minutes?

   (c) (10 points) When will the mass be 80 grams? (You may leave ln(...) in your answer)

Figure 1: Problem 8

9. (30 points) Find the solutions of the logistic equation $y' = y(10 - y)$ with initial condition $y(0) = 1$ and another with $y(2) = 1$.