(1) Find the tangent spaces of the following surfaces:
   (a) \( S = \{x^3 + y^3 + z^3 = 10\} \subset \mathbb{R}^3 \) at \((1, 1, 2)\).
   (b) \( S = \{(u^2, uv, v^2) : u, v \in \mathbb{R}\} \subset \mathbb{R}^3 \) at \((1, 2, 4)\).

(2) (MPDC p. 88 Ex. 4) Show that the tangent planes of a surface given by \( z = xf(y/x) \) for \( x \neq 0 \) and a differentiable function \( f \) all pass through the origin \((0, 0, 0)\).

(3) (MPDC p. 89 Ex. 7) Let \( S \) be a regular surface in \( \mathbb{R}^3 \) and let \( f : S \to \mathbb{R} \) be the map given by \( f(p) = ||p - p_0||^2 \) for a fixed point \( p_0 \in \mathbb{R}^3 \). Show that \( df_p(w) = 2w.(p - p_0) \) for all \( p \in S \) and \( w \in T_{S,p} \).

(4) (MPDC p. 89 Ex. 8) Prove that if \( L : \mathbb{R}^3 \to \mathbb{R}^3 \) is a linear map and \( S \subset \mathbb{R}^3 \) is a regular surface invariant under \( L \), i.e., \( L(S) \subset S \), then the restriction \( L|_S \) is a differential map and \( dL_p(w) = L(w) \) for all \( p \in S \) and \( w \in T_{S,p} \).

(5) (MPDC p. 89 Ex. 10) Let \( \alpha : I \to \mathbb{R}^3 \) be a regular curve parameterized by its arc length with nonzero curvature everywhere. Let

\[
  f(s, v) = \alpha(s) + r(n(s) \cos v + b(s) \sin v), \quad \text{for } r \equiv \text{const} \neq 0 \text{ and } s \in I,
\]

be a parameterized surface in \( \mathbb{R}^3 \), where \( n(s) \) and \( b(s) \) are the normal and binormal vectors of \( \alpha \), respectively. Show that when \( f \) is regular, its unit normal vector is

\[
  N(s, v) = -(n(s) \cos v + b(s) \sin v).
\]