Math 348 Assignment #2
Due Sept. 25, 2015

(1) Let $\alpha : I \to \mathbb{R}^m$ and $\beta : I \to \mathbb{R}^n$ be two $C^m$-curves. Show that
\[ \frac{d^m}{dt^m} (\alpha(t), \beta(t)) = \sum_{j=0}^{m} \left( \begin{array}{c} m \\ j \end{array} \right) \alpha^{(j)}(t) \beta^{(m-j)}(t). \]

(2) A regular smooth curve $\gamma : I \to \mathbb{R}^n$ has zero curvature on $I$ if and only if it is a line.

(3) A regular smooth curve $\gamma : I \to \mathbb{R}^2$ has nonzero constant curvature on $I$ if and only if it is an arc of a circle. Hint: Show that $\gamma(s) = (x(s), y(s))$, parameterized by its arc length, satisfies
\[ \left\{ \begin{array}{ll} kx'(s) = y''(s) \\
y'(s) = -x''(s) \end{array} \right. \text{ or } \left\{ \begin{array}{ll} kx'(s) = -y''(s) \\
y'(s) = x''(s) \end{array} \right. \]

(4) (MPDC p. 25 Ex. 11) For a smooth regular plane curve given by $\rho = \rho(\theta)$ under the polar coordinates ($\rho, \theta$), do the following:
   (a) Show that its arc length is
   \[ \int_a^b \sqrt{(\rho(\theta))^2 + (\rho'(\theta))^2} d\theta \]
   from $\theta = a$ to $\theta = b$.
   (b) Show that its curvature is
   \[ \kappa(\theta) = \frac{|2(\rho')^2 - \rho \rho'' + \rho^2|}{((\rho')^2 + \rho^2)^{3/2}}. \]

(5) Find the curvature of the curve
\[ \{x^2 + y^2 - z^2 = x + y + z + 2 = 0\} \subset \mathbb{R}^3 \]
at the points $(0, -1, -1)$ and $(-3, -4, 5)$, respectively.