(1) Prove that an affine set is a convex set.

(2) In $\mathbb{R}^4$, let $x_1 = (1, -1, 2, -1)$, $x_2 = (2, -1, 2, 0)$, $x_3 = (1, 0, 2, 0)$ and $x_4 = (1, 0, 3, 1)$.
   (a) Show that the set $\{x_1, x_2, x_3, x_4\}$ is affinely independent.
   (b) Let $A = \text{aff}\{x_1, x_2, x_3, x_4\}$ and $B = \{(z_1, z_2, z_3, z_4) : z_1 + z_2 + z_3 - z_4 = 3\}$. Show that $A = B$.

(3) Prove the following variation of Caratheodory’s Theorem: Let $S$ be a nonempty subset of $\mathbb{R}^n$, let $v \in S$ and let $x \in \text{conv}(S)$. Then there exist points $x_1, x_2, ..., x_n$ in $S$ such that $x \in \text{conv}\{v, x_1, x_2, ..., x_n\}$.

(4) Prove or give a counterexample: If $A$ and $B$ are nonempty convex sets with $A \subset B$, then $\text{relint}(A) \subset \text{relint}(B)$.

(5) Let $S$ be a closed set with $\text{Int}(S) \neq \emptyset$. If $x \not\in S$ and $y \in \text{Int}(S)$. Prove that there exists a point $z \in \text{relint} xy$ such that $z \in \text{bd}(S)$.

(6) Let $S$ be a convex set. If $x \in \text{Int}(S)$ and $y \in \text{cl}(S)$, then $\text{relint} xy \subset \text{Int}(S)$.

(7) Let $S$ be a convex set with $\text{Int}(S) \neq \emptyset$. Prove that $\text{cl}(\text{Int}(S)) = \text{cl}(S)$, and find an example to show that the convexity of $S$ is necessary.

(8) Let $S$ be a convex set with $\text{Int}(S) \neq \emptyset$. Prove that $\text{Int}(\text{cl}(S)) = \text{Int}(S)$, and find an example to show that the convexity of $S$ is necessary.

(9) Let $S$ be a convex set. Prove that $\text{bd}(\text{cl}(S)) = \text{bd}(S)$, and find an example to show that the convexity of $S$ is necessary.

(10) Let $S$ be a closed convex set and $x, y \in \text{bd}(S)$. Prove that either $xy \subset \text{bd}(S)$ or $\text{relint} xy \subset \text{Int}(S)$.