(1) (a) Since 
\[(A + B) + C = \{a + b : a \in A, b \in B\} + \{c : c \in C\}\]
and
\[A + (B + C) = \{a : a \in A\} + \{b + c : b \in B, c \in C\}\]
it follows that \((A + B) + C = A + (B + C)\).

(b) Since 
\[\alpha(A + B) = \alpha\{a + b : a \in A, b \in B\} = \{\alpha a + \alpha b : a \in A, b \in B\}\]
and
\[\alpha A + \alpha B = \{\alpha a : a \in A\} + \{\alpha b : b \in B\} = \{\alpha a + \alpha b : a \in A, b \in B\}\]
it follows that \(\alpha(A + B) = \alpha A + \alpha B\).

(2) Since \(A_1 = \{(x, 0) : 0 \leq x \leq 2\}\) and \(A_2 = \{(0, y) : 0 \leq y \leq 2\}\), \(A_1 + A_2 = \{(x, y) : 0 \leq x, y \leq 2\}\),
\[A_1 + A_4 = \bigcup_{u \in A_1} (u + A_4) = \{(x, y) : -1 < y < 1, -\sqrt{1-y^2} < x < 2 + \sqrt{1-y^2}\}\]
and
\[(A_1 + A_2) + A_3 = \bigcup_{u \in A_3} (u + (A_1 + A_2)) = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq 4, x-2 \leq y \leq x+2\}\]

(3) (a) Let \(S = B(p, \delta)\) be the open ball centered at point \(p\) with radius \(\delta\). We want to show that \(S\) is open. Let \(q \in S\). Then \(\varepsilon = d(p, q) < \delta\). We claim \(B(q, \delta - \varepsilon) \subset S\). Let \(r \in B(q, \delta - \varepsilon)\). Then \(d(q, r) < \delta - \varepsilon\). By the triangle inequality,
\[d(p, r) \leq d(p, q) + d(q, r) < \varepsilon + (\delta - \varepsilon) = \delta\]
Therefore, \(r \in B(p, \delta) = S\) and hence \(B(q, \delta - \varepsilon) \subset S\). Consequently, \(q\) is an interior point of \(S\) and \(S\) is open.

(b) Every point of \(\mathbb{R}^n\) is obviously an interior point of \(\mathbb{R}^n\). So \(\mathbb{R}^n\) is open.

(c) Let \(f : \mathbb{R}^n \to \mathbb{R}\) be the function \(f(x_1, x_2, ..., x_n) = x_1 + x_2 + ... + x_n\). Since \(f\) is continuous, \(f^{-1}(U)\) is open for every open set \(U \subset \mathbb{R}\). Since \(V = \{(x_1, x_2, ..., x_n) : x_1 + x_2 + ... + x_n \neq 0\}\) is open in \(\mathbb{R}\), \(V\) is consequently open.

(4) (a) Let \(\{U_i : i \in I\}\) and \(\{V_j : j \in J\}\) be the collections of open sets contained in \(A\) and \(B\), respectively. Since \(A \subset B\), \(\{U_i\}_{i \in I} \subset \{V_j\}_{j \in J}\) and hence \(\text{Int}(A) = \bigcup_{U_i \in I} U_i \subset \bigcup_{V_j \in J} V_j = \text{Int}(B)\).

(b) Let \(\{Y_i \supset A : i \in I\}\) and \(\{Z_j \supset B : j \in J\}\) be the collections of close sets containing \(A\) and \(B\), respectively. Since \(A \subset B\), \(\{Y_i\}_{i \in I} \supset \{Z_j\}_{j \in J}\) and hence \(\text{cl}(A) = \bigcap_{Y_i \in I} Y_i \subset \bigcap_{Z_j \in J} Z_j = \text{cl}(B)\).

(c) Let \(\{Y_i \supset A : i \in I\}\) and \(\{Z_j \supset B : j \in J\}\) be the collections of affine sets containing \(A\) and \(B\), respectively. Since \(A \subset B\), \(\{Y_i\}_{i \in I} \supset \{Z_j\}_{j \in J}\) and hence \(\text{aff}(A) = \bigcap_{Y_i \in I} Y_i \subset \bigcap_{Z_j \in J} Z_j = \text{aff}(B)\).
For any \( y \in B(x, r) \), \( \|\lambda y - \lambda x\| = \lambda \|y - x\| \leq \lambda r \). Therefore, \( \lambda y \in B(\lambda x, \lambda r) \) and
\[
\lambda B(x, r) \subset B(\lambda x, \lambda r).
\]
Similarly, we can prove
\[
\frac{1}{\lambda} B(\lambda x, \lambda r) \subset B(x, r)
\]
and hence \( \lambda B(x, r) \supset B(\lambda x, \lambda r) \). Therefore, \( \lambda B(x, r) = B(\lambda x, \lambda r) \).
\[
(a) \quad \text{Since } A, \text{ then } A \text{ is open because } \text{Int}(A) \text{ is. On the other hand, if } A \text{ is open, every point of } A \text{ is an interior point and hence } A = \text{Int}(A).
\]
\[
(b) \quad \text{If } A = \text{cl}(A), \text{ then } A \text{ is closed because } \text{cl}(A) \text{ is. On the other hand, if } A \text{ is closed, } A \subset A \text{ and hence } \text{cl}(A) \subset A; \text{ and since } A \subset \text{cl}(A), A = \text{cl}(A).
\]
\[
(c) \quad \text{If } A = \text{aff}(A), \text{ then } A \text{ is affine because } \text{aff}(A) \text{ is. On the other hand, if } A \text{ is affine, } A \subset A \text{ and hence } \text{aff}(A) \subset A; \text{ and since } A \subset \text{aff}(A), A = \text{aff}(A).
\]
\[
(d) \quad \text{Since } \text{cl}(A) \text{ is closed, } \text{cl}(\text{cl}(A)) = \text{cl}(A) \text{ by (c).}
\]
\[
(f) \quad \text{Since } \text{Int}(A) \text{ is open, } \text{Int}(\text{Int}(A)) = \text{Int}(A) \text{ by (b).}
\]
\[
(g) \quad \text{Since } \text{aff}(A) \text{ is affine, } \text{aff}(\text{aff}(A)) = \text{aff}(A) \text{ by (d).}
\]
(7) Let \( V = \{ p_1, p_2, ..., p_n \} \) be a finite set. We want to show that \( V \) is closed, i.e., the complement \( V^c \) is open. Let \( p \in V^c \) and let \( r_i = d(p, p_i) \). Since \( p \notin V \), \( r_i > 0 \) for each \( i \). Let \( r = \min(r_1, r_2, ..., r_n) \). Then \( B(p, r) \cap V = \emptyset \) and hence \( B(p, r) \subset V^c \). So \( V^c \) is open and \( V \) is closed. We also need to show that \( V \) is bounded. Let \( R = \max_{1 \leq i \leq n} d(o, p_i) + 1 \), where \( o \) is the origin. Then \( p_i \in B(o, R) \) for \( i = 1, 2, ..., n \) and hence \( V \subset B(o, R) \) is bounded. Therefore, \( V \) is compact.

(a) Since \( A + B = \bigcup_{b \in B} (A + b) \) and \( A + b \supseteq A \) is open since the union of a collection of open sets is open.

(b) This is false. Let \( A = \{ m : m \in \mathbb{Z}, m > 0 \} \) and \( B = \{ 1/n - n : n \in \mathbb{N}, n > 0 \} \). So \( A + B = \{ m - n + 1/n : m, n \in \mathbb{Z}, n > 0 \} \).

Since \( \lim_{n \to -\infty} n = \infty \), \( A \) is closed; similarly, since \( \lim_{n \to \infty} (1/n - n) = -\infty \), \( B \) is closed.

We will show that \( 0 \in \text{cl}(A + B) \), \( 0 \notin A + B \) and hence \( A + B \) is not closed.

Since \( n - n + 1/n = 1/n \in A + B \) and \( \lim_{n \to \infty} 1/n = 0 \), \( 0 \in \text{cl}(A + B) \). We want to show that \( 0 \notin A + B \). Suppose that \( 0 \in A + B \). Then \( m - n + 1/n = 0 \) for some \( m, n > 0 \) and \( m, n \in \mathbb{Z} \). Hence \( m = n - 1/n \in \mathbb{Z} \) and we necessarily have \( n = 1 \). But then \( m = 0 \), which is impossible.

(a) Suppose that \( x_1, x_2, ..., x_n \) are affinely dependent. Then there exist \( \lambda_1, \lambda_2, ..., \lambda_n \in \mathbb{R}, \) not all zero, such that \( \sum_{i=1}^n \lambda_i = 0 \) and \( \sum_{i=1}^n \lambda_i x_i = 0 \).

Since \( \lambda_1 = - (\lambda_2 + \lambda_3 + ... + \lambda_n) \),
\[
- (\lambda_2 + \lambda_3 + ... + \lambda_n) x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n = 0
\]
\[
\Rightarrow \lambda_2(x_2 - x_1) + \lambda_3(x_3 - x_1) + ... + \lambda_n(x_n - x_1) = 0
\]
Obviously, \( \lambda_2, \lambda_3, ..., \lambda_n \) cannot be all zero; otherwise, \( \lambda_1 = 0 \) and all \( \lambda_i \)'s are zero. Consequently, \( x_1 - x_2, x_1 - x_3, ..., x_1 - x_n \) are linearly dependent.

Suppose that \( x_1 - x_2, x_1 - x_3, ..., x_1 - x_n \) are linearly dependent. Then there exist \( \lambda_2, \lambda_3, ..., \lambda_n \in \mathbb{R}, \) not all zero, such that \( \sum_{i=2}^n \lambda_i (x_1 - x_i) = 0 \).

Consequently,
\[
-(\lambda_2 + \lambda_3 + ... + \lambda_n) x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_n x_n = 0
\]
Since \(-(\lambda_2 + \lambda_3 + \ldots + \lambda_n) + \lambda_2 + \lambda_3 + \ldots + \lambda_n = 0\) and \(\lambda_2, \lambda_3, \ldots, \lambda_n\) are not all zero, \(x_1, x_2, \ldots, x_n\) are linearly dependent.

(b) Suppose that \(x_1, x_2, \ldots, x_n\) are linearly dependent. Then there exist \(\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R}\), not all zero, \(\sum_{i=1}^{n} \lambda_i x_i = 0\). Without the loss of generality, we may assume that \(\lambda_1 \neq 0\). Then \(x_1 = -\sum_{i=2}^{n} (\lambda_i/\lambda_1) x_i\) is a linear combination of \(x_2, x_3, \ldots, x_n\).

On the other hand, suppose that one of \(x_i\)'s is a linear combination of the rest. Without the loss of generality, suppose that \(x_1 = \sum_{i=2}^{n} \lambda_i x_i\) is a linear combination of \(x_2, x_3, \ldots, x_n\). Then \(x_1 - \lambda_2 x_2 - \lambda_3 x_3 - \ldots - \lambda_n x_n = 0\) and hence \(x_1, x_2, \ldots, x_n\) are linearly dependent.

(c) Suppose that \(x_1, x_2, \ldots, x_n\) are affinely dependent. Then there exist \(\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{R}\), not all zero, such that \(\sum_{i=1}^{n} \lambda_i = 0\) and \(\sum_{i=1}^{n} \lambda_i x_i = 0\). Without the loss of generality, we may assume that \(\lambda_1 \neq 0\). Then \(x_1 = -\sum_{i=2}^{n} (\lambda_i/\lambda_1) x_i\) is an affine combination of \(x_2, x_3, \ldots, x_n\). Since \(-\sum_{i=2}^{n} (\lambda_i/\lambda_1) = 1\).

On the other hand, suppose that one of \(x_i\)'s is an affine combination of the rest. Without the loss of generality, suppose that \(x_1 = \sum_{i=2}^{n} \lambda_i x_i\) is an affine combination of \(x_2, x_3, \ldots, x_n\) with \(\sum_{i=2}^{n} \lambda_i = 1\). Then \(x_1 - \lambda_2 x_2 - \lambda_3 x_3 - \ldots - \lambda_n x_n = 0\) and hence \(x_1, x_2, \ldots, x_n\) are affinely dependent since \(1 = \sum_{i=2}^{n} \lambda_i = 0\).

(10) Choose \(x \in F \subset G\). Then \(W = F - x\) and \(V = G - x\) are linear subspaces. Since \(F \subset G\), \(W \subset V\). And since \(\dim W = \dim V = k\), \(W = V\) and hence \(F = G\).