XI CHEN	Math 341 Midterm	Feb 27, 2003

PRINT NAME:____

STUDENT ID NUMBER:________________________________

(1) No books, notes and calculators are allowed.
(2) Show your work in details.

Problem	Points	Score
1	20	
2	20	
3	15	
4	20	
5	25	
Total	100	

(1) (20 points)

(a) Prove that $x_1, x_2, ..., x_k$ are affinely dependent if and only if $x_2 - x_1, x_3 - x_1, ..., x_k - x_1$ are linearly dependent.

(b) Prove that $x_1, x_2, ..., x_k$ are affinely dependent if and only if one of the x_i's is the affine combination of the others.

(2) (20 points) Let $A, B \subset \mathbb{R}^n$.

(a) Prove that if one of A and B is open, $A + B$ is open.

(b) Prove that if A and B are convex, $A + B$ is convex.

(3) (15 points) Let $A \subset B \subset \mathbb{R}^n$. Prove the following:

(a) $\mathrm{cl}(A) \subset \mathrm{cl}(B)$

(b) $\mathrm{int}(A) \subset \mathrm{int}(B)$

(c) $\mathrm{conv}(A) \subset \mathrm{conv}(B)$

(4) (20 points) Let F and G be flats. Prove the following:

(a) If $\dim F = \dim G$ and $F \subset G$, then $F = G$.

(b) If $F \cup G$ is convex, then either $F \subset G$ or $G \subset F$.

(5) (25 points) Let S be a convex set with $\text{int}(S) \neq \emptyset$. Prove that $\text{int}(\text{cl}(S)) = \text{int}(S)$ and find an example to show that the convexity of S is necessary.

Hint: You may prove it along the following line of argument:

(a) Pick a point $y \in \text{int}(S)$. Let $x \in \text{int}(\text{cl}(S))$. There exists an open ball $B(x, \delta) \subset \text{cl}(S)$.

(b) Let $r = d(x, y)$ and

$$z = \frac{2r + \delta}{2r} x - \frac{\delta}{2r} y$$

Show that $z \in B(x, \delta)$ and $x \in \overline{yz}$.

(c) Use the following fact to finish the proof: if S is convex, $y \in \text{int}(S)$ and $z \in \text{cl}(S)$, then $\text{relint}(\overline{yz}) \subset \text{int}(S)$.