PRINT NAME: ____________________________________________

STUDENT ID NUMBER: __________________________________

(1) No books, notes and calculators are allowed.
(2) Show your work in details.

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(1) (20 points)

(a) Prove that $x_1, x_2, \ldots, x_k$ are affinely dependent if and only if $x_2 - x_1, x_3 - x_1, \ldots, x_k - x_1$ are linearly dependent.

(b) Prove that $x_1, x_2, \ldots, x_k$ are affinely dependent if and only if one of the $x_i$’s is the affine combination of the others.
(2) (20 points) Let $A, B \subset \mathbb{R}^n$.

(a) Prove that if one of $A$ and $B$ is open, $A + B$ is open.

(b) Prove that if $A$ and $B$ are convex, $A + B$ is convex.
Let $A \subset B \subset \mathbb{R}^n$. Prove the following:

(a) $\text{cl}(A) \subset \text{cl}(B)$

(b) $\text{int}(A) \subset \text{int}(B)$

(c) $\text{conv}(A) \subset \text{conv}(B)$
(4) (20 points) Let $F$ and $G$ be flats. Prove the following:

(a) If $\dim F = \dim G$ and $F \subset G$, then $F = G$.

(b) If $F \cup G$ is convex, then either $F \subset G$ or $G \subset F$. 
(5) (25 points) Let \( S \) be a convex set with \( \text{int}(S) \neq \emptyset \). Prove that \( \text{int}(\text{cl}(S)) = \text{int}(S) \) and find an example to show that the convexity of \( S \) is necessary.

Hint: You may prove it along the following line of argument:
(a) Pick a point \( y \in \text{int}(S) \). Let \( x \in \text{int}(\text{cl}(S)) \). There exists an open ball \( B(x, \delta) \subset \text{cl}(S) \).
(b) Let \( r = d(x, y) \) and

\[
z = \frac{2r + \delta}{2r} x - \frac{\delta}{2r} y
\]

Show that \( z \in B(x, \delta) \) and \( x \in \overline{yz} \).
(c) Use the following fact to finish the proof: if \( S \) is convex, \( y \in \text{int}(S) \) and \( z \in \text{cl}(S) \), then \( \text{relint}(\overline{yz}) \subset \text{int}(S) \).