Math 341 Homework 6 (due Mar. 13)

Problems from the book: (p. 33, 40) 3.8, 3.10, 3.11, 3.14, 4.1, 4.2, 4.3, 4.5, 4.6

Hints

3.10 (a) Use 3.9. Since $\text{aff}(S)$ is closed, $\text{cl}(\text{aff}(S)) = \text{aff}(S)$. Since $S \subset \text{aff}(S)$, $\text{cl}(S) \subset \text{cl}(\text{aff}(S))$. (b) similar.

(c) Use 2.20. Let $y \in \text{aff}(S)$ and $x \in \text{relint}(S)$. Then $\text{relint}\,\overline{xy} \cap \text{relint}(S) \neq \emptyset$ by 2.20. Let $z \in \text{relint}\,\overline{xy} \cap \text{relint}(S)$. Then y is an affine combination of z and x. Hence $y \in \text{aff}(\text{relint}(S))$.

3.11 H_1 and H_2 are parallel to each other if $H_2 = x_0 + H_1$ for some x_0.

4.1 By Theorem 4.12, $\text{conv}(B)$ cannot be closed. So we are looking for a closed set B such that $\text{conv}(B)$ is not closed. Take $B = \{y = x^2, x \geq 0\}$. Then $\text{conv}(B) = \{y \geq x^2, x > 0\} \cup \{(0,0)\}$. Let $A = \{x = 0, 1 \leq y \leq 2\}$. Verify that $\text{conv}(A) \cap \text{conv}(B) = \emptyset$ but A and B cannot be strictly separated by a line.

4.2 Use Theorem 4.12.

4.3 Use 3.14 to show that $\pi(S)$ is convex. To show that $\pi(S)$ is relative open in G, prove first the fact that $\pi(B(x, \delta)) = B(\pi(x), \delta)$.

4.6 By Theorem 4.7, A and B are separated by a hyperplane H. To show that H actually strictly separates A and B, you have to show that $H \cap A = H \cap B = \emptyset$.