Math 341 Homework 5 (due Mar. 6)

Problems from the book: (p. 25, 32, 33) 2.37, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.9

Hints

2.37 For convex sets enclosed by exactly one convex set, think of the following: Let $B = \{x^2 + y^2 \leq 1\}$ be a disk and p be a point on the circle $\mathrm{bd}(B) = \{x^2 + y^2 = 1\}$. Show that $S = B \backslash \{p\}$ is enclosed by exactly one convex set (B obviously). Use the same idea to contruct sets enclosed by exactly n convex sets for every $n \geq 0$. The sequence in (b) can be similarly contructed.

(c) is true. The proof goes as follows: Suppose that there exist two points $p, q \in A$ and $p, q \notin B$. Then $q \in \mathrm{conv}(B \cup \{p\})$ and $p \in \mathrm{conv}(B \cup \{q\})$ (Why? It is your job to justify it). Then $p \in \overline{xq}$ and $q \in \overline{py}$ for some $x, y \in B$ (Again, justify it). Then $p, q \in \overline{xy}$ (draw a picture show where x, y, p, q are and you will see why this is true).

(d) is false. Let x_1, x_2, x_3 be three points on $\mathbb{R}^2$ not lying on a line and $\Delta x_1 x_2 x_3$ be the triangle with vertices at x_1, x_2, x_3 (including the boundary). Let $B = \Delta x_1 x_2 x_3 \backslash \overline{x_1 x_2}$, $p \in \overline{x_1 x_2}$ and $p \neq x_1, x_2$. Show that $B \cup \{p\}$ is not convex.