Math 341 Homework 3 (due Feb. 6)

Problems from the book: (pp. 23-24) 2.13, 2.14, 2.15, 2.16, 2.21, 2.24, 2.27, 2.28, 2.29

Hints

2.13 Use 2.12. Fix a point $y \in \text{int}(S)$. For a point $x \in \text{cl}(S)$, $\text{relint}(\overline{xy}) \subset \text{int}(S)$. So $x \in \text{cl}(\text{int}(S))$. For the counterexample, consider $S = \{2\} \cup (0,1) \subset \mathbb{R}$ (you may use this counterexample but I encourage you to come up with your own and it is not hard at all).

2.14 Use 2.13. Let $A = \text{int}(\text{cl}(S))$ and $B = \text{int}(S)$. Then by 2.13, $\text{cl}(A) = \text{cl}(B)$. Prove the following: if A and B are two open convex sets with $\text{cl}(A) = \text{cl}(B)$, then $A = B$. Again use 2.12. For the counterexample, consider $S = \mathbb{Q}$ the set of rational numbers.

2.15 Follows directly from 2.14 using the fact $\text{bd}(A) = \text{cl}(A) \setminus \text{int}(A)$. You may use the same conterexample as in 2.14.

2.16 For the conterexample, consider $S = \{2\} \cup (0,1)$ and $\alpha = \beta = 1$. Show that $S + S = (0,2) \cup (2,3) \cup \{4\}$ and $2S = (0,2) \cup \{4\}$.