

Math 341 Homework 2 (due Jan. 30)

Problems from the book: (pp. 22-23) 2.6, 2.7, 2.8, 2.9, 2.10, 2.11, 2.12, 2.19

Additional problems:

- A1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function with $f''(x) > 0$ everywhere (i.e. $f(x)$ is concave upward). Show that the convex hull of $\{y = f(x)\}$ in \mathbb{R}^2 is the set $\{y \geq f(x)\}$.
- A2. What is the convex hull of $\{y = f(x)\}$ if $f(x)$ is a function with $f''(x) < 0$ (i.e. $f(x)$ is concave downward)?

Hints

2.7 (f) Change $-\pi/2 < x < \pi/2$ to $-\pi/2 < y < \pi/2$. It is a misprint IMO.

2.11 Change $x \in \text{bd}(S)$ to $z \in \text{bd}(S)$. It is an obvious typo.

2.19 (b) Let $S = \{x > 0\} \subset \mathbb{R}^2$. Show that S is convex and the boundary of S is also convex.

A1. Prove the fact that the graph of $y = f(x)$ is always above any of its tangent lines. You need mean value theorem.

A2. Consider $y = -f(x)$, which is concave upward.