A2.1 Let \( S = \{1, 2, \ldots, n\} \), \( V = \{ f : S \to \mathbb{R} \} \) be the vector space of all functions \( f : S \to \mathbb{R} \) and \( T : V \to \mathbb{R}^n \) be the map 
\[
T(f) = (f(1), f(2), \ldots, f(n)).
\]
Show that \( T \) is an invertible linear transformation.

A2.2 Let \( T_1 : \mathbb{R}^2 \to \mathbb{R}^2 \) and \( T_2 : \mathbb{R}^2 \to \mathbb{R}^2 \) be two linear endomorphisms satisfying \( T_1 \circ T_2 = T_2 \circ T_1 = 0 \), 
\[
T_1(3, 2) = (1, 0) \quad \text{and} \quad T_2(1, 1) = (2, 1).
\]
(a) Find \( T_1 \) and \( T_2 \).
(b) Find the kernels and ranges of \( T_1 \) and \( T_2 \).

A2.3 We call a bijection \( s : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\} \) a permutation of \( \{1, 2, \ldots, n\} \). Let \( V \) be a vector space with an ordered basis 
\[
B = \{v_1, v_2, \ldots, v_n\},
\]
s be a permutation of \( \{1, 2, \ldots, n\} \) and \( C \) be another ordered basis of \( V \) given by 
\[
C = \{v_{s(1)}, v_{s(2)}, \ldots, v_{s(n)}\}.
\]
Show that the change-of-coordinate matrix \( P_{C \leftarrow B} \) is 
\[
P_{C \leftarrow B} = [e_{s^{-1}(1)} \ e_{s^{-1}(2)} \ \cdots \ e_{s^{-1}(n)}]
\]
where \( s^{-1} \) is the inverse map of \( s : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\} \) and \( e_1, e_2, \ldots, e_n \) are the column vectors of \( I_n \). Also find \( P_{B \leftarrow C} \).

A2.4 Let \( T : \mathbb{R}[x] \to \mathbb{R}[x] \) be the map given by 
\[
T(f(x)) = 2f(x) - f(1 - x).
\]
Show that \( T \) is an invertible linear endomorphism and find \( T^{-1} \). Hint: Let \( g(x) = 2f(x) - f(1 - x) \), substitute \( x \) by \( 1 - x \) and then solve it for \( f(x) \).

A2.5 Let \( W, W_1, W_2, W_3, \ldots, W_n \) be \( n + 1 \) subspaces of a vector space \( V \). Show that if 
\[
W \subset W_1 \cup W_2 \cup W_3 \cup \ldots \cup W_n,
\]
then \( W \) is contained in one of \( W_1, W_2, W_3, \ldots, W_n \). You just have to prove this for \( n = 2 \) but gain +20 bonus points for all \( n \). Hint
for all $n$: prove it by induction on $n$ and first show that $W$ is contained in one of

\begin{align*}
W_2 \cup W_3 \cup \ldots \cup W_n \\
W_1 \cup W_3 \cup \ldots \cup W_n \\
\vdots \\
W_1 \cup W_2 \cup \ldots \cup W_{n-1}
\end{align*}