A1.1 Verify Cayley-Hamilton Theorem for $2 \times 2$ matrices: show that

$$A^2 - \text{tr}(A)A + \det(A)I = 0$$

for every $2 \times 2$ matrix $A$.

A1.2 Let $S$ be a set and $V = \{f : S \to \mathbb{R}\}$ be the set of all functions $f : S \to \mathbb{R}$. Show that $V$ is a vector space over $\mathbb{R}$ with vector addition and scalar multiplication defined by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) \text{ and } (cf)(x) = cf(x)$$

for $f_1, f_2, f \in V$ and $c \in \mathbb{R}$.

A1.3 In A1.2, let $S = \{1, 2, ..., n\}$ and let $V$ be the set of all functions $f : S \to \mathbb{R}$. Find a basis for $V$ and $\dim V$. Justify your answer.

A1.4 Let $V$ be a vector space over $\mathbb{C}$. Show that if $\{v_1, v_2, ..., v_n\}$ is a basis of $V$ over $\mathbb{C}$, then $\{v_1, v_2, ..., v_n, iv_1, iv_2, ..., iv_n\}$ is a basis of $V$ over $\mathbb{R}$ and thus conclude that $\dim_{\mathbb{R}} V = 2 \dim_{\mathbb{C}} V$.

A1.5 Find the intersection $V_1 \cap V_2$ and the sum $V_1 + V_2$ of the subspaces $V_1$ and $V_2$ of $\mathbb{R}^4$ for

(a) $V_1 = \{x_1 - x_2 = 0\}$ and $V_2 = \{x_3 - x_4 = 0\}$;
(b) $V_1 = \{(-t, t, -t, 0)\}$ and $V_2 = \{x_1 + x_2 = x_3 + x_4 = 0\}$.