(1) Reduce each of the following square matrices $A$ to its Jordan Canonical Form. That is, find an invertible matrix $P$ such that $P^{-1}AP$ is a Jordan matrix.

\[ \begin{bmatrix} 4 & -1 \\ 9 & -2 \end{bmatrix} \quad \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \]

(2) Let $J_{\lambda,n}$ be the $n \times n$ Jordan block

\[ J_{\lambda,n} = \begin{bmatrix} \lambda & 1 & & \\ & \lambda & 1 & \\ & & \ddots & \ddots \\ & & & \lambda & 1 \end{bmatrix}_{n \times n} \]

with eigenvalue $\lambda$. Do the following:

(a) Find the characteristic polynomial and eigenvectors of $J_{\lambda,n}$.
(b) Show that

\[ (J_{\lambda,n} - \lambda I)^k \mathbf{e}_i = \mathbf{e}_{i-k} \]

for all nonnegative integers $k$ and $i$, where $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\}$ is the standard basis of $\mathbb{R}^n$ and we set $\mathbf{e}_i = 0$ for $i \leq 0$.
(c) Show that

\[ (J_{\lambda,n} - \lambda I)^n = 0. \]

(3) Which of the following statements are true and which are false? Justify your answer.

(a) For a linear endomorphism $T : V \to V$, if $\mathbf{v} \in K(T^3)$, then $T(\mathbf{v}) \in K(T^2)$ and $T^2(\mathbf{v}) \in K(T)$.
(b) The sum of two eigenvectors of a square matrix $A$ must also be an eigenvector of $A$.
(c) For two similar matrices $A$ and $B$,

\[ \text{Nul}(A - \lambda I) = \text{Nul}(B - \lambda I) \]

for all $\lambda$.
(d) For every $2 \times 2$ real matrix $A$, $\{I, A, A^2\}$ is always linearly dependent over $\mathbb{R}$. 


(4) List all $4 \times 4$ Jordan matrices with the only eigenvalue 1. How many $n \times n$ Jordan matrices are there with the only eigenvalue 1? Justify your answer.

(5) Let $T_1 : V \to V$ and $T_2 : V \to V$ be two linear endomorphisms on a vector space $V$ satisfying $T_1 \circ T_2 = T_2 \circ T_1$.

(a) Show that 
$$K(T_1) + K(T_2) \subset K(T_1 \circ T_2).$$

(b) Let $W = K(T_1 \circ T_2)$. Show that 
$$T_1(W) \subset W \text{ and } T_2(W) \subset W.$$  

(c) Suppose that $W = K(T_1 \circ T_2)$ is finite-dimensional and 
$K(T_1) \cap K(T_2) = \{0\}$. Show that 
$$K(T_1) + K(T_2) = K(T_1 \circ T_2).$$