Math 311 Midterm Review

Some information on the midterm:

- Time and location: 9:00-9:50, Oct. 25, 2013, CAB
- Sections covered by the midterm: (BMPS) 1.0-1.5, 2.1-2.4, 3.1-3.2, 3.4-3.5

A sample midterm:

(1) Determine where each of the following functions \( f(z) \) is holomorphic and find its complex derivative \( f'(z) \) where it is holomorphic. You must justify your answer.
   (a) \( f(z) = x +yi \), where \( x = \text{Re}(z) \) and \( y = \text{Im}(z) \).
   (b) \( f(z) = z^2 \).
   (c) \( f(z) = 2\sin(z) \).
   (d) \( f(z) = \text{Log}(1 - 2z) \), where \( \text{Log} \) is the principal branch of \( \log z \).

(2) Show that
   \[ 5 \sinh(|y|) \leq |3 \cos z + 4 \sin z| \leq 5 \cosh(y) \]
   for all complex numbers \( z \), where \( y = \text{Im}(z) \).

(3) Find all the complex roots of the equation
   \[ \cos z + \sin z = 2. \]

(4) Let
   \[ T(z) = \frac{i - z}{i + z}. \]
   and \( D = \{|z + 2| < 1\} \). Find and sketch the image \( T(D) \).

(5) Let \( f : D \to \mathbb{C} \) be a complex function defined in the disk
   \( D = \{|z-z_0| < r\} \). Show that if \( f(z) \) is differentiable at \( z_0 \) and
   \( f'(z_0) \neq 0 \), then \( f(z) \) is not differentiable at \( z_0 \).