(1) Find the four roots of the polynomial $z^4 + 4$ and use these to factor $z^4 + 4$ into two quadratic polynomials with real coefficients.

(2) Write the following functions $f(z)$ in the Cartesian form

$$f(z) = u(x, y) + iv(x, y)$$

with $u(x, y) = \text{Re}(f(z))$ and $v(x, y) = \text{Im}(f(z))$:

(a) $f(z) = z^2 + z + i$;
(b) $f(z) = \frac{z + 1}{i - z}$;
(c) $f(z) = \exp(z^2 + z)$;
(d) $f(z) = 2z + \bar{z}$.

(3) Is it true that $\overline{f(z)} = f(z)$ for all complex functions $f : \mathbb{C} \to \mathbb{C}$ and all $z \in \mathbb{C}$? Prove it if yes and give a counterexample if no.

(4) Let $f(z)$ and $g(z)$ be two complex functions defined on $G \subset \mathbb{C}$. Suppose that

$$|f(z) + g(z)| < |f(z)| + |g(z)|$$

for all $z \in G$. Show that

(a) $f(z) \neq 0$ and $g(z) \neq 0$ for every $z \in G$.
(b) The image of $G$ under the map $h(z) = f(z)/g(z)$ satisfies

$$h(G) \subset \mathbb{C}\setminus[0, \infty)$$

where $\mathbb{C}\setminus[0, \infty) = \mathbb{C}\setminus\{x + yi : x \geq 0, y = 0\}$.

(5) Let $T(z) = z/(z + 1)$. Find the images and inverse images of the following sets under $T$:

(a) $|z| < 1/2$;
(b) $\text{Re}(z) = 1/2$.

(6) Compute the following limits if they exist:

(a) $\lim_{z \to i} \frac{iz^3 - 1}{z^2 + 1}$;
(b) $\lim_{z \to 0} \frac{\text{Im}(z)^2}{z}$.
(c) \( \lim_{z \to 0} \frac{\text{Im}(z^2)}{z} \);
(d) \( \lim_{z \to 0} e^{-1/z^2} \).

(7) Let \( \text{Log}(z) \) be the principal branch of \( \log(z) \). Compute the following:
(a) \( \log(\pi + \pi i) \);
(b) \( \text{Log}(\pi - \pi i) \);
(c) \( \log(e^{\pi + \pi i}) \);
(d) \( \text{Log}(e^{\pi - \pi i}) \).

(8) Let
\[ f(z) = \exp\left(\frac{1}{3} \text{Log}(z)\right) \]
be the principal branch of \( \sqrt[3]{z} \). Show that \( f(z) \) is not continuous everywhere on \( \mathbb{C}^* = \mathbb{C}\{0\} \). Find the largest set \( D \subset \mathbb{C}^* \) where \( f(z) \) is continuous.

(9) Let \( f(z) \) be the principal branch of \( \sqrt[3]{z} \). Is it true that
\[ f(z_1 z_2) = f(z_1) f(z_2) \]
for all \( z_1, z_2 \in \mathbb{C}^* \)? Prove it if yes and give a counterexample if no.

(10) Sketch the following sets in the complex plane \( \mathbb{C} \) and determine whether they are open, closed, or neither; bounded; connected. Briefly state your reason.
(a) \( |z + 3| > 1 \);
(b) \( |\text{Re}(z)| \geq 1 \);
(c) \( 1 \leq |z + 3| < 2 \);
(d) \( |z| > |z + 1| \).