Math 311 Assignment #1
Due Sept 13, 2013

(1) Show that
   (a) \( \text{Re}(iz) = \frac{z - \bar{z}}{2i} \)
   (b) \( \text{Im}(iz) = \frac{z + \bar{z}}{2} \)
for all complex numbers \( z \).

(2) Verify the law of distribution for multiplication of complex numbers. That is, show that
   \( z_1(z_2 + z_3) = z_1z_2 + z_1z_3 \)
for all \( z_1, z_2, z_3 \in \mathbb{C} \).

(3) Compute
   (a) \( \frac{4 + i}{5 - i} \)
   (b) \( (\sqrt{3} - i)^4 \)

(4) Let \( f \) be the map sending each complex number
   \[ z = x + yi \to \begin{bmatrix} x & y \\ -y & x \end{bmatrix} \]
Show that \( f(z^{-1}) = (f(z))^{-1} \) for all complex numbers \( z \neq 0 \).

(5) Write the following complex numbers in polar form:
   (a) \( 1 - i \)
   (b) \( 2i \)
   (c) \( 1 + \sqrt{3}i \)
   (d) \( \sqrt{3} - i \)

(6) Write the following complex numbers in rectangular form:
   (a) \( 2 \exp(2\pi i/3) \)
   (b) \( 3i \)
   (c) \( \exp(2013\pi i/4) \)
   (d) \( 5\pi i \)

(7) Let \( \Delta ABC \) be a triangle in the complex plane whose vertices \( A, B, C \) are given by \( a, b, c \in \mathbb{C} \). Show that the area of \( \Delta ABC \) is given by
   \[ S_{\Delta ABC} = \frac{1}{2} | \text{Im}((a - b)(a - c)) |. \]
(8) Let $\Delta ABC$ be a triangle in the complex plane and $O$ be its center. Show that

$$S_{\Delta OAB} = S_{\Delta OBC} = S_{\Delta OCA}.$$

(9) Compute $(1 + i)^{2013}$.

(10) Use binomial theorem

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n = \sum_{k=0}^{n} \binom{n}{k}a^{n-k}b^k$$

to show that

$$\sum_{k=0}^{1006} (-1)^k \binom{2013}{2k} = \binom{2013}{0} - \binom{2013}{2} + \binom{2013}{4} - \ldots + \binom{2013}{2012} = -2^{1006}$$