Math 311 Assignment #4
Due Oct. 11, 2011

(1) Show that if \( f(z) \) is continuous at \( z_0 \), so is \( |f(z)| \).

(2) Let
\[
f(z) = \begin{cases} \frac{z^3}{z^2} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}
\]

Show that
(a) \( f(z) \) is continuous everywhere on \( \mathbb{C} \);
(b) the complex derivative \( f'(0) \) does not exist.

(3) Show that \( f(z) \) in (2) is actually nowhere differentiable, i.e., the complex derivative \( f'(z) \) does not exist for any \( z \in \mathbb{C} \).

(4) Find \( f'(z) \) when
(a) \( f(z) = z^2 - 4z + 2 \);
(b) \( f(z) = (1 - z^2)^4 \);
(c) \( f(z) = \frac{z + 1}{2z + 1} (z \neq -\frac{1}{2}) \);
(d) \( f(z) = e^{1/z} (z \neq 0) \).

(5) Prove the following version of complex L’Hospital: Let \( f(z) \) and \( g(z) \) be two complex functions defined on \( |z - z_0| < r \) for some \( r > 0 \). Suppose that \( f(z_0) = g(z_0) = 0 \), \( f(z) \) and \( g(z) \) are differentiable at \( z_0 \) and \( g'(z_0) \neq 0 \). Then
\[
\lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{f'(z_0)}{g'(z_0)}
\]

(6) Show that if \( f(z) \) satisfies the Cauchy-Riemann equations at \( z_0 \), so does \( (f(z))^n \) for every positive integer \( n \).