(1) Let $C$ denote the circle $|z| = 1$ oriented counterclockwise.
   (a) Show that
   \[ \int_C z^n \exp \left( \frac{1}{z} \right) \, dz = \frac{2\pi i}{(n + 1)!} \]
   for $n = 0, 1, 2, \ldots$.
   (b) Show that
   \[ \int_C \exp \left( z + \frac{1}{z} \right) \, dz = 2\pi i \sum_{n=0}^{\infty} \frac{1}{n!(n + 1)!} \].

(2) Let the degrees of the polynomials
   
   \[ P(z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_n z^n (a_n \neq 0) \]
   
   and
   
   \[ Q(z) = b_0 + b_1 z + b_2 z^2 + \ldots + b_m z^m (b_m \neq 0) \]
   
   be such that $m \geq n + 2$. Show that if all the zeros of $Q(z)$ are
   interior to a simple closed contour $C$, then
   \[ \int_C \frac{P(z)}{Q(z)} \, dz = 0. \]

(3) Write the principal part of each of the following functions at
    its isolated singular point and determine whether the point is a
    pole, a removable singular point, or an essential singular point:
    (a) $z \exp \left( \frac{1}{z} \right)$;
    (b) $\sin \frac{z}{z}$;
    (c) $\frac{1}{(2 - z)^3}$.

(4) Show that the singular point of each of the following functions
    is a pole. Determine the order of the pole and the residue of
    the function at the pole.
    (a) $\frac{1 - \cosh z}{z^3}$;
    (b) $\frac{1 - e^{2z}}{z^4}$;
\begin{align*}
\text{(c) } & \frac{e^{2z}}{(z-1)^2}.
\end{align*}

(5) Find the value of the integral
\[
\int_C \frac{3z^3 + 2}{(z - 1)^2(z^2 + 9)} \, dz
\]
 taken counterclockwise around the circle (a) \(|z - 2| = 2\) (b) \(|z| = 4\).

(6) Let \(C_N\) denote the boundary of the square whose edges lie along the lines
\[x = \pm \left( N + \frac{1}{2}\right) \pi \text{ and } y = \pm \left( N + \frac{1}{2}\right) \pi\]
 oriented counterclockwise, where \(N\) is a positive integer.
(a) Show that
\[
\int_{C_N} \frac{dz}{z^2 \sin z} = 2\pi i \left( \frac{1}{6} + 2 \sum_{n=1}^{N} \frac{(-1)^n}{n^2 \pi^2} \right).
\]
(b) Show that
\[
\lim_{N \to \infty} \int_{C_N} \frac{dz}{z^2 \sin z} = 0.
\]
(c) Show that
\[
\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.
\]