(1) By differentiating the Taylor series representation
\[ \frac{1}{1 - z} = \sum_{n=0}^{\infty} z^n \ (|z| < 1) \]

obtain the expansions
\[ \frac{1}{(1 - z)^2} = \sum_{n=0}^{\infty} (n + 1)z^n \ (|z| < 1) \]
and
\[ \frac{1}{(1 - z)^3} = \sum_{n=0}^{\infty} \frac{(n + 1)(n + 2)}{2} z^n \ (|z| < 1). \]

(2) By substituting \( \frac{1}{1 - z} \) for \( z \) in the expansion
\[ \frac{1}{(1 - z)^2} = \sum_{n=0}^{\infty} (n + 1)z^n \ (|z| < 1) \]
derive the Laurent series representation
\[ \frac{1}{z^2} = \sum_{n=2}^{\infty} \frac{(-1)^n(n - 1)}{(z - 1)^n} \ (1 < |z - 1| < \infty). \]

(3) Find the Taylor series expansions of \( \cos z \) at \( z = \pm \pi/2 \). Use this to prove that if
\[ f(z) = \begin{cases} 
\cos z & \text{when } z \neq \pm \pi/2, \\
\frac{-1}{\pi} & \text{when } z = \pm \pi/2,
\end{cases} \]
then \( f(z) \) is an entire function.

(4) Prove that if \( f \) is analytic at \( z_0 \) and \( f(z_0) = f'(z_0) = ... = f^{(m)}(z_0) = 0 \), then the function \( g \) defined by means of the equations
\[ g(z) = \begin{cases} 
f(z) & \text{when } z \neq z_0, \\
\frac{f(z)}{(z - z_0)^{m+1}} & \text{when } z = z_0
\end{cases} \]
is analytic at \( z_0 \).
(5) Use multiplication of series show that
\[ \frac{e^z}{z(z^2 + 1)} = \frac{1}{z} + 1 - \frac{1}{2}z^2 - \frac{5}{6}z^4 + ... \]
for 0 < |z| < 1.

(6) Use division to obtain the Laurent series representation
\[ \frac{1}{e^z - 1} = \frac{1}{z} - \frac{1}{2} + \frac{1}{12}z - \frac{1}{720}z^3 + ... \]
for 0 < |z| < 2\pi.

(7) Find the residue at z = 0 of the function
(a) \( \frac{1}{z + z^2} \);
(b) \( z \cos \left( \frac{1}{z} \right) \);
(c) \( \frac{\sinh z}{z^4(1 - z^2)} \).

(8) Evaluate the integral of each of these functions around the circle |z| = 3 oriented counterclockwise:
(a) \( \frac{\exp(-z)}{z^2} \);
(b) \( \frac{z + 1}{z^2 - 2z} \).