A1. Compute
\[ \int_0^\infty \frac{\cos x}{(1 + x^2)^2} \, dx \]

A2. Let \( f(z) = z^3 + a_1z^2 + a_2z + a_3 \) be a cubic polynomial with three distinct roots \( r_1, r_2, r_3 \).
(a) Use residue theorem to compute
\[ \int_{|z|=R} \frac{dz}{f(z)} \]
where the circle \( |z| = R \) is oriented counter-clockwise and \( R > |r_1|, |r_2|, |r_3| \).
(b) Show that
\[ \lim_{R \to \infty} \int_{|z|=R} \frac{dz}{f(z)} = 0 \]
(c) Combine (1) and (2) to show the identity
\[ \frac{1}{(r_1 - r_2)(r_1 - r_3)} + \frac{1}{(r_2 - r_3)(r_2 - r_1)} + \frac{1}{(r_3 - r_1)(r_3 - r_2)} = 0 \]

A3. Find all the Laurent expansions of \( f(z) = 1 + \frac{z^4}{z^2(z-1)^2} \) about \( z = 2 \).

A4. Let \( f(z) \) be an analytic function on \( 0 < |z-p| < r \). We say that \( f(z) \) has a zero at \( p \) of order \( n \) if \( f(z) = (z-p)^ng(z) \) for some analytic function \( g(z) \) on \( |z-p| < r \) with \( g(p) \neq 0 \); and we say that \( f(z) \) has a pole at \( p \) of order \( n \) if \( f(z) = (z-p)^{-n}g(z) \) for some analytic function \( g(z) \) on \( |z-p| < r \) with \( g(p) \neq 0 \). Show that if \( f(z) \) has a zero or pole at \( p \) of order \( n > 0 \), then
\[ \frac{f'(z)}{f(z)} \]
has a pole at \( p \) of order 1; and
\[ \text{Res}_p \left( \frac{f'(z)}{f(z)} \right) = \begin{cases} n & \text{if } f(z) \text{ has a zero at } p \\ -n & \text{if } f(z) \text{ has a pole at } p \end{cases} \]

A5. Compute
\[ \int_{|z|=2006} \frac{1}{e^z - 1} \, dz \]
where the circle \( |z| = 2006 \) is oriented counter-clockwise.