Problems from the book:

**Sec. 5.** (p. 62): 4

**Sec. 6.** (p. 73): 1, 4

Additional problems:

**A1.** We know that if either

\[
\lim_{n \to \infty} \frac{|a_n|}{a_{n+1}} = R
\]

or

\[
\lim_{n \to \infty} \frac{1}{\sqrt{|a_n|}} = R
\]

the radius of convergence of the power series \( \sum_{n=0}^{\infty} a_n z^n \) is \( R \). Give an example of a power series \( \sum_{n=0}^{\infty} a_n z^n \) such that

- \( a_n \neq 0 \) for all \( n \);
- neither of the above limits exists;
- the power series have radius of convergence 1.

**A2.** Let

\[
f(z) = \frac{1}{1 - z - z^2}
\]

and let

\[
a_n = \frac{f^{(n)}(0)}{n!}
\]

for \( n = 0, 1, 2, \ldots \). Show that \( a_{n+2} = a_{n+1} + a_n \) for all \( n \).