A5.1 Let $A$ and $B$ be two similar $n \times n$ matrices. Suppose that $B = P^{-1}AP$ for some invertible matrix $P$. Show that if $v(t)$ is a solution of the system of linear ODEs
\[ \frac{dx}{dt} = Ax \]
then $P^{-1}v(t)$ is a solution of
\[ \frac{dx}{dt} = Bx, \]
where
\[ v(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}. \]

A5.2 Let $W_1$ and $W_2$ be two subspaces of $\mathbb{R}^4$ given by
\[ W_1 = \text{Span}\{(1, 1, 0, 0), (1, 0, 1, 0)\} \] and
\[ W_2 = \{(x_1, x_2, x_3, x_4) : x_1 - x_2 = x_3 + x_4 = 0\}. \]
Do the following:
(a) Find bases for $W_1^\perp$, $W_2^\perp$ and $(W_1 + W_2)^\perp$, respectively.
(b) Let $v = (1, -1, 1, 1)$. Find the projections of $v$ onto $W_1, W_2$ and $W_1 + W_2$, respectively.

A5.3 Orthogonally diagonalize and find spectral decompositions of the following real symmetric matrices:
\[ a) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad b) \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \]

A5.4 Let $A$ be a $3 \times 3$ real symmetric matrix with characteristic polynomial $(x - 1)(x - 2)^2$ and
\[ \text{Nul}(A - 2I) = \text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}. \]
Find $A$. 

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