(1) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation with characteristic polynomial $x^4 - 3x^2 + 2$. Find the characteristic polynomial of $T^3$.

(2) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by
\[ T(x, y) = (3x + 4y, 4x - 3y). \]
Find $T^n$ and $T^{2014}(1, 2)$.

(3) Let $V$ be the vector space of real polynomials of degree $\leq n$ and $T : V \to V$ be the linear transformation given by
\[ T(f(x)) = f(2x + 1). \]
Is $T$ diagonalizable? If it is, find a basis $B$ of $V$ such that $[T]_{B,B}$ is diagonal.

(4) Which of the following statements are true and which are false? Justify your answer.

(a) Let $A$ and $B$ be two $n \times n$ matrices. If there exist $n$ linearly independent vectors $v_1, v_2, ..., v_n \in \mathbb{R}^n$ that are the eigenvectors of both $A$ and $B$, then $AB = BA$.

(b) Two $2 \times 2$ matrices with the same characteristic polynomial must be similar.

(c) Let $V$ be a vector space of finite dimension and let $T_1 : V \to V$ and $T_2 : V \to V$ be two linear transformations. If both $T_1$ and $T_2$ are diagonalizable, so is $T_1 + T_2$.

(d) Let $V$ be a vector space of finite dimension and let $T_1 : V \to V$ and $T_2 : V \to V$ be two linear transformations. If both $T_1$ and $T_2$ are diagonalizable, so is $T_1 \circ T_2$.

(5) Let $V$ be the vector space of real polynomials of degree $\leq 3$ and $T : V \to V$ be the linear transformation given by
\[ T(f(x)) = f(x) + f'(x). \]
Find $T^n$ and $T^{2014}(x^3)$.

(6) Let $\{a_n : n = 0, 1, 2, ...\}$ be a sequence given by
\[ a_n = 2a_{n-1} + a_{n-2} + 1 \]
for all $n \geq 2$ and $a_0 = 1$ and $a_1 = 4$. Find a formula for $a_n$. 

(7) Let $V$ be a real vector space of finite dimension and $T : V \rightarrow V$ be a linear transformation satisfying $T^2 = T + I$. Show that $T$ is diagonalizable.

(8) Let $T : V \rightarrow V$ be a linear transformation with the property that every nonzero vector $v \in V$ is an eigenvector of $T$. Show that $T(v) \equiv cv$ for some constant $c$. 